DOCUMENT RESUME

ED 356 431 CE 063 551

AUTHOR Leidig, Julie; And Others
TITLE The Competitive Edge: Sharpening Your Skills in the Workplace. Mathematics. Instructor's Guide.
INSTITUTION Texas Univ., Austin. Extension Instruction and Materials Center.
SPONS AGENCY Office of Vocational and Adult Education (ED), Washington, DC. National Workplace Literacy Program.
REPORT NO SN-150-I
PUB DATE 93
NOTE 316p.; For related documents, see CE 063 548-552.
AVAILABLE FROM Extension Instruction and Materials Center, P.O. Box 7218, University of Texas, Austin, TX 78713-7218 (stock no. SN-150-I: $15; set of five, $52; 100 or more, 10% discount).
PUB TYPE Guides - Classroom Use - Teaching Guides (For Teacher) (052)

EDRS PRICE MF01/PC13 Plus Postage.
DESCRIPTORS Adult Basic Education; Basic Skills; Course Content; *Education Work Relationship; Learning Activities; Learning Modules; Lesson Plans; *Mathematics Instruction; *Remedial Mathematics; Teaching Methods
IDENTIFIERS *Workplace Literacy

ABSTRACT

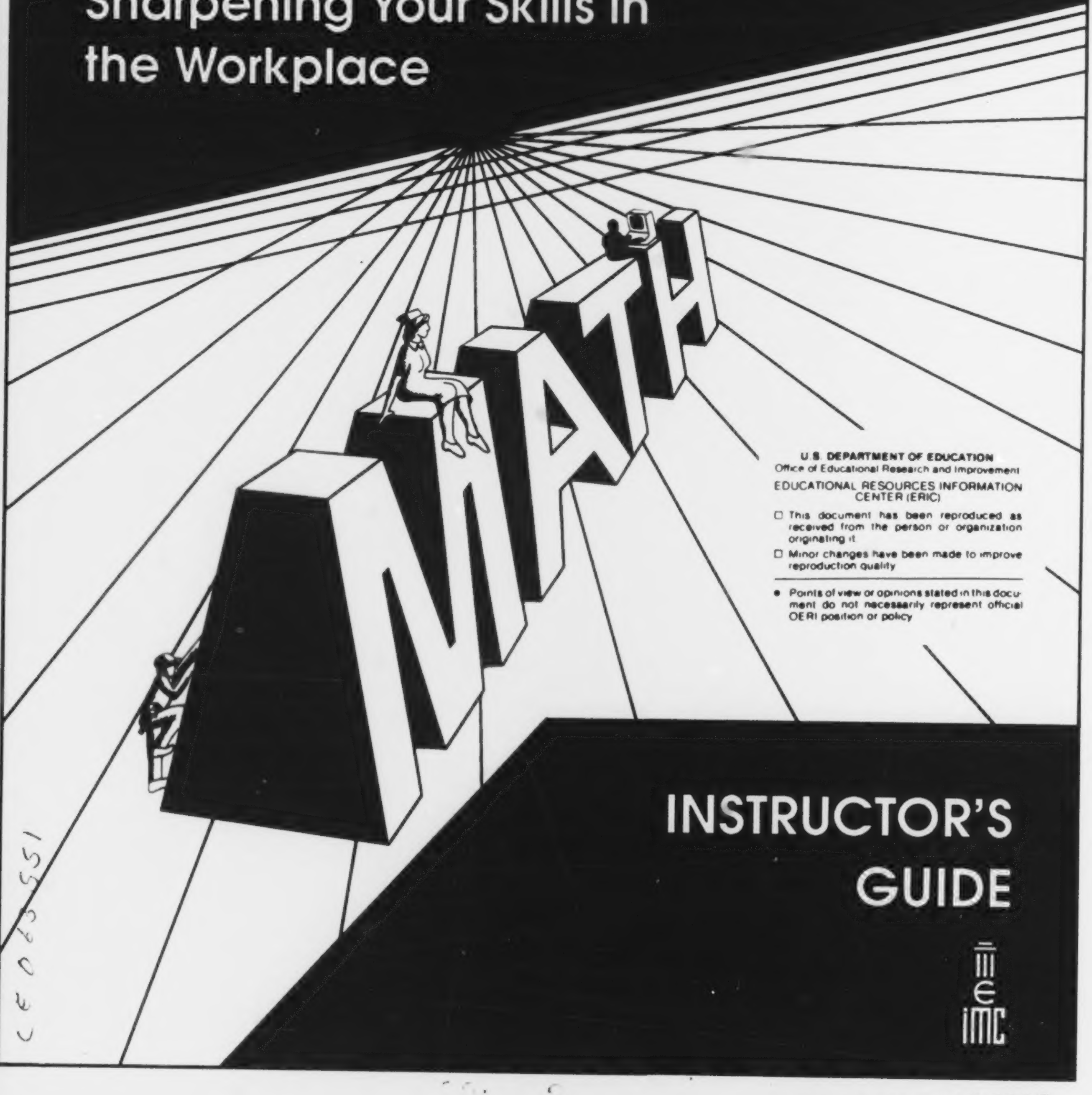
This instructor's guide to mathematics instruction is one of the five books developed during a 21-month cooperative project to develop instructional materials that enhance skills in a workplace context. Partners in the project included the University of Texas at Austin, consultants, the Austin/Travis County Private Industry Council, Hart Graphics, IBM, and Texas Instruments. The guide contains two modules that cover the following topics for a suggested 20-week course in refresher mathematics: decimals, fractions, word problems, estimating and averages, ratios and proportions, percents, standard measurement, signed numbers, algebraic equations and inequalities, solving for unknowns, working with formulas, and graphing ordered pairs. The modules are divided into a total of 23 lessons that use a functional workplace context, discovered through observing how workers use skills, conducting job task analyses, and interviewing workers, supervisors, and training managers. Lessons in this instructor's guide contain some or all the following elements of a lesson plan: learning objective; introduction; vocabulary; practice problems; developing understanding; skill building problems, activities, and exercises; self-check; suggestions for further activities, discussion, or practice; suggested lesson supplements from those at the back of the module; pretests and posttests; answer keys; and notes to the instructor (preceding each lesson). (KC)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED356431

THE Competitive Edge

Sharpening Your Skills in the Workplace



U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
☐ This document has been reproduced as received from the person or organization originating it
☐ Minor changes have been made to improve reproduction quality
● Points of view or opinions stated in this document do not necessarily represent official OERI position or policy

INSTRUCTOR'S GUIDE

iMC

CE 063 551

BEST COPY AVAILABLE

The Competitive Edge: Sharpening Your Skills in the Workplace

MATHEMATICS

Instructor's Guide

By

Julie Leidig
Sally McCarty
Holly O'Rear
Mary Hartzell

Coordinated by

Elaine Shelton

Published by

The University of Texas at Austin
Educational Resources
Extension Instruction and Materials Center
Division of Continuing Education

1993

© 1993 by The University of Texas at Austin
All rights reserved. Published 1993
Printed in the United States of America

Brand names, company names, trademarks, or other identifying symbols appearing in the illustrations and/or text are used to clarify and identify and do not constitute a recommendation or endorsement by the authors or the publisher.

The contents of this book were developed under a grant from the Department of Education. However, those contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

Editor / Barbara Sutherland
Artists / Elinor Gamache and Debra Dionisio
Publishing Technician / Kathryn N. Roberts
Production Assistant / Debbie Hajji
Production Coordinator / Barbara Sutherland

Additional copies may be ordered from:

THE UNIVERSITY OF TEXAS AT AUSTIN
Educational Resources
Extension Instruction and Materials Center
P.O. Box 7218
Austin, TX 78713-7218
512-471-7716 or 1-800-252-3461 (within Texas)
FAX 512-471-7853

Stock Number SN 150 I

Contents

Acknowledgments v

The Inside Scoop ix

MODULE 1 PRETEST

MODULE 1: Refresher Math (Part 1)

Lesson 1.1: Adding and Subtracting Decimals 1

Lesson 1.2: Multiplying Decimals 9

Lesson 1.3: Dividing with Decimals 15

Lesson 1.4: Using Fractions: Adding, Subtracting, and Reducing to Lowest Terms 21

Lesson 1.5: Finding Common Denominators 29

Lesson 1.6: Multiplying and Dividing Fractions 39

Lesson 1.7: Fractions and Decimals 47

MODULE 1 POSTTEST

MODULE 2 PRETEST

MODULE 2: Refresher Math (Part 2)

Lesson 2.1: Solving Word Problems 53

Lesson 2.2: Estimating and Averaging 65

Lesson 2.3: Working with Ratios 75

Lesson 2.4: Working with Proportions 87

Lesson 2.5: Converting Fractions and Decimals into Percents 97

Lesson 2.6: Solving Percent Problems 105

REVIEW #1 113

Lesson 2.7: Standard Measurement (Part 1) 119

Lesson 2.8: Standard Measurement (Part 2) 133

Lesson 2.9: Metric Measurement 143

Lesson 2.10: Adding and Subtracting Signed Numbers 155

Lesson 2.11: Multiplying and Dividing Signed Numbers 165

REVIEW #2 171

Lesson 2.12: Algebraic Equations and Inequalities 177

Lesson 2.13: Addition and Subtraction, Solving for an Unknown 193

Lesson 2.14: Multiplication and Division, Solving for an Unknown 201

Lesson 2.15: Working with Formulas 213

Lesson 2.16: Graphing Ordered Pairs 223

REVIEW #3 239

MODULE 2 POSTTEST

Answer Key for Pretests and Posttests

Acknowledgments

The Competitive Edge: Sharpening Your Skills in the Workplace is the final product of a 21-month National Workplace Education grant from the U.S. Department of Education to the Extension Instruction and Materials Center at The University of Texas at Austin, in cooperation with its business partner, the Austin/Travis County Private Industry Council. The project, titled *Skills Building for Tomorrow*, had two main objectives. The first was to develop instructional materials that enhance skills in a workplace context. The second objective was to field-test these materials while providing a yearlong instructional program at three companies in Austin: Hart Graphics, IBM, and Texas Instruments. During the 21 months of this project, we worked with the following individuals whom we gratefully acknowledge for their valuable contributions to the success of this endeavor.

The University of Texas at Austin, Extension Instruction and Materials Center, Educational Resources, Skills Building for Tomorrow Project
Dr. Kathryn Allen, Director, Extension Instruction and Materials Center
Dr. Barbara Selke-Kern, Director, Educational Resources
Elaine Shelton, Director, Skills Building for Tomorrow Project (SBT)
Julie Leidig, SBT Instructor Coordinator/Curriculum Developer/Assistant Editor
Sally McCarty, SBT Instructor/Curriculum Developer
Holly O'Rear, SBT Instructor/Curriculum Developer
Mary Hartzell, SBT Instructor/Curriculum Developer
Cathy Crandall, Coordinator of Customer Service

Consultants
Sue Lagerquist, Instructional Consultant
Kurt von Quintus, Technical Consultant

Austin/Travis County Private Industry Council
Bill Demestihas, Executive Director

Hart Graphics
Mary Astor
Lola Barkley
Cheryl Bingham
Orlando Boleda
David Camp
Donald Castilleja
Daniel Castillo
Domisco Castillo
Peggy Coker
Shakir Doctor
Kevin Duffy
Charles Ferraro
Sandra Fisher
Gregory Galloway
Jim Gipson
Donna Harris
David Hart
Joe Hernandez
Everett Holbrook
Robert Kirby
Robert Luna
Olivia Machuca
Larry Maynard
Emilio Mendez
Jamie Millard
Albird Mindieta
Rosie Molina
T. J. Morris
Edwin Muniz
Paul Noel
Brian Oetzel
Dan Pogue

Patrick Randolph
Dois Ripley
Tom Savage
David Schoen
Dario Solis
Sue Syphers
Scott Thomas
James Waddington
Daniel Walker
Dennis Weisser
Sandra Washington

IBM
Melinda Acevedo
Bertha O. Alvarez
Jesse Ancira
Joyce Arnold
A. C. Berry
Bill Black
Sandy Boccio
Karl Borman
Bob Borron
Linda E. Brooks
Ronnie D. Cain
Patrick J. Camacho
Ted Carillo
Mack Carmona
Rosa Castro
Sandra Clark
Robert R. Cornejo
Robert Ebert
M. Louise Felps
Dennis Ferguson
Joe Fiol
Jennifer Garcia
Tommy Green
Margaret Guerrero
Ray Gutierrez
Davis Hallmark
Deon Hector
Aaron A. Hendricks
Lydia G. Hinojosa
Pete Holmes
Vera Houston
Lynda Howard
Tommy Ivarra, Jr.
Shirley M. Jackson
David Johnson
Helena Jones
David LaFuente
Patsy E. Lewis
Alex Lopez, Jr.
Delores Martinez
Victor Martinez
Wendell McBride
Judy McCaffery
Irene Mendez
Ralph Mendoza
Jerry Miller
Larry Charles Ockletree, Sr.
Rollie L. Olson
Rance F. Parrott
Susan Patnaude
Barbara Perez
Yolanda Perez
Betty Perry
Pat Ramirez
Virginia Reyes
Josie R. Rodriguez
Virginia Ross
Andrea Michelle Ryan
Greg Sharp
Tony Skubiata
Michael K. Steedley
Gayland Thorn
Joe G. Valdez
Alfred Vasquez
Mack Wilson
Wanda Wood
Wally Young
Sam Zigrossi

Texas Instruments
Tim Andrews
Linda Anguiano
Gaye Arnold
Juanita Becker
Don Block
Barbara Bloom
Berneice Bonnet
Jake Boyd
Greg Briceno
Bill Caley
Fran Carpenter
Tony Collins
Roberta Cook
Angie Cosler
Gilbert Costilla
Gary Costons
Mike Cuellar
Glen Davis
Mel Day
Cheri Deba
Frank De La Rosa
Gwen Delaune
Thereatha Donnell
Virginia Dujka
Mike Duncan
Kelly Eastham
Leland Eaves
Maria Estrada
Carolyn Erwin
Carolyn Ewing
Teresa Fenrich
Paul Fonseca
Linda Geesling
Frank Gonzales
Gloria Grant
Diana Griffith
Bob Halper
Marty Herrin
George Hendrix
Albert Hernandez
Willie Ingle
Gerald Johnson
Nora Johnson
Nancy Jokovich
Nancy Jones
Dee Karunasena
Don Karunasena
Eddie Lebriton
Elvia Lopez
Jesse Lozoria

Fran Martin
Andy Martinez
Rick Masselink
Steve Melchor
Brad Mertz
Gloria Moon
Dave Morris
Isabel Murrieta
Alwin Neidig
Lucy Nixon
Geraldine Pool
Jackie Pool
Matt Priestley
Erma Pruneda
Jo Redding
Alice Reemsnyder
Sharon Ricker
Debbie Rivers
Minnie Rodriguez
Howard Rogers
Joe Salazar
Somboun Siriboury
Howard Swathwood
Rungston Titapas
Warida Trejo
Pat Watson
Freddie West
Bob Wright
Sook Yoo

The Inside Scoop

Employers want employees who can keep up with changing technology and an ever-changing work environment. Employees want to obtain and maintain the necessary skills that make them valuable to the company. Workplace education strengthens a range of skills that are valuable in the daily business lives of employers and employees. The goal of these lessons is to enhance the intermediate-level skills of workers to those of functional competence.

Overview of Books

The Competitive Edge: Sharpening Your Skills in the Workplace consists of five books:

- Mathematics, Student Edition
- Mathematics, Instructor's Guide
- Communications, Student Edition
- Communications, Instructor's Guide
- Administrator's Guide

The *Mathematics* and *Communications* books are separated into modules. The suggested time guidelines are approximately 19 weeks for *Mathematics* and 22 weeks for *Communications*. By module, it would be as follows:

Mathematics
Module 1: Refresher Math (Part 1) = 6 weeks
Module 2: Refresher Math (Part 2) = 13 weeks

Communications
Module 1: Accessing Information Through Reading = 10 weeks
Module 2: Decision Making, Problem Solving, and Team Building = 5 weeks
Module 3: Oral and Written Communication = 7 weeks

The actual teaching time for each module depends upon students' characteristics, readiness, and speed of mastery as well as company requirements. The two math modules can be taught in sequence, or you can alternate between a math module and a communications module. Most lessons take about 2 hours to cover, but some may take more or less time.

Lesson Design

The lessons were designed using a **functional workplace context** by observing how workers actually use skills. This process included conducting job task analyses and interviewing workers, supervisors, and training managers.

This curriculum provides instruction in the specific skills that employees use in the workplace. The lessons in each module build on one another and are designed to be taught together as a unit. Each lesson includes several sections. Icons signal the beginning of each section. Not all lessons will include every section. The icons used in the lessons and in the memos to the instructor are as follows:

Learning Objective

The learning objective is given at the beginning of each lesson. For example: "To develop tools for sending clear verbal messages."

Introduction

The introduction restates the objective and often mentions its relevance to the workplace. For example:

"When work teams solve problems, they are seeking new, more effective results. Something about the old way is not working. Team members need to think creatively to find new solutions. This lesson will focus on attitudes and tools that can help you respond creatively to solving problems."

Vocabulary

The vocabulary section defines terms that may need explanation. For example:

"*scanning*—using key words or numbers to look quickly through a reading passage for specific information."

Now Try This

A few practice problems are presented at certain progressive stages in the development of the lesson. The learners can practice their understanding of the concepts at each stage. For example:

"Round off 9.212 to the nearest whole number."

Develop Your Understanding

This section contains the concepts and skills being taught. Explanations and examples are found in this section.

Skill Builders

Math problems, activities, and other exercises for student practice are located in this section.

Check Yourself

This section, at the end of the lesson, can include a variety of math problems, activities, or exercises for extra practice.

Lesson Break

This is a signal to the instructor that this lesson contains many concepts. The broken pencil suggests to the instructor that this may be a good place to "break" and check for mastery.

Closure

Suggestions for further activities, discussion, or practice are located in this section.

Tools of the Trade

This is a signal that you can find lesson supplements at the back of the module. Lesson supplements include practical reviews on topics like language mechanics, dictionary usage, and spelling tips. They are located after Modules 1 and 3 in the *Communications* book only.

Answer keys are provided at the back of the student editions and within the lessons of the instructor's guides. Some of the questions in the *Communications* book are open-ended and do not have one correct answer. Use your discretion when judging the correctness of answers to those questions.

The *Math* instructor's guide includes pretests and posttests for both modules, with answers at the back. Module 2 in the *Math* book has three Reviews.

The pages in the instructor's guides are numbered to exactly correspond to the numbering of the student editions.

There is a "Memo" to the instructor preceding each lesson that will alert you to special problems, suggestions, or explanations.

Suggestions to the Instructor

Many of the examples used in the lessons are from the manufacturing and/or high-tech world of work. These examples can easily be adapted to incorporate situations that are applicable to other company-specific contexts. By all means, use actual company memos, reports, newsletters, charts, etc., to make the examples and activities even more directly relevant to your learners.

You may choose to omit some lessons and extend others depending on class and company needs. Present the material in stages that are logical and easy to follow.

Review both the student lesson and the "Memo" to the instructor before each session. Prepare materials or examples that make the lesson content applicable to the workplace. By using material specific to that business, the lessons can be adapted to almost any work environment. To reinforce past learning, review the previous lesson at the beginning of each new session.

Ask learners to comment on how they might use each skill in previous, current, or future jobs. Encourage them to seek opportunities to apply skills on the job.

Encourage student involvement by developing a nonthreatening environment. Vary the structure of the class with individual and group exercises, games, and various applications.

Conclusion

By sharpening these workplace skills, each employee-learner has an opportunity to increase his/her value to the company. And each employer, in turn, has an increased opportunity to maintain and, in some cases reclaim, its competitive edge in our global economy. You are playing a key role in facilitating such positive growth on both an individual and a global level.

NAME ______________________________

Module 1 Pretest

Directions: *Do your calculations in the space provided.*

1. Which of the following numbers is the largest?

   a. 15.379
   b. 15.397
   c. 15.244
   d. 15.563

2. Which of the following numbers is the smallest?

   a. 6.54
   b. 6.45
   c. 6.25
   d. 6.89

3. Round off the decimal number 5.9 to a whole number.

4. Add: 3 + 0.24 + 2.8 =

5. Add: 1.045 + 2.306 + 62.1 =

6. Subtract: 14 – 1.4 =

7. Subtract: 17.001 – 14.984 =

8. Multiply: 821.7 × 12 =

9. Multiply: 6.385 × .02 =

10. Divide: $1.4\,\overline{)1.75}$

11. Divide: $0.05\,\overline{)629}$

12. Change the improper fraction to a mixed number.

$\frac{35}{8} =$

13. Change the mixed number to an improper fraction.

$1\frac{3}{7} =$

Reduce *all* fractions to lowest terms in items 14 through 17.

14. Add: $\frac{1}{5} + \frac{3}{5} =$

15. Add: $\frac{2}{7} + \frac{6}{7} =$

16. Subtract: $\frac{7}{9} - \frac{4}{9} =$

17. Subtract: $\frac{5}{6} - \frac{3}{6} =$

18. What is the least common denominator for $\frac{3}{4}$ and $\frac{1}{3}$?

19. What is the least common denominator for $\frac{3}{8}$ and $\frac{2}{4}$?

Add and simplify items 20 and 21.

20. Add: $\frac{3}{4} + \frac{1}{5} =$

21. Add: $7\frac{1}{4} + 8\frac{3}{4} =$

Subtract and simplify items 22 and 23.

22. Subtract: $\frac{2}{3} - \frac{1}{9} =$

23. Subtract: $1\frac{3}{5} - \frac{1}{3} =$

Multiply and simplify items 24 and 25.

24. Multiply: $\frac{2}{5} \times 5 =$

25. Multiply: $\frac{1}{2} \times \frac{3}{4} =$

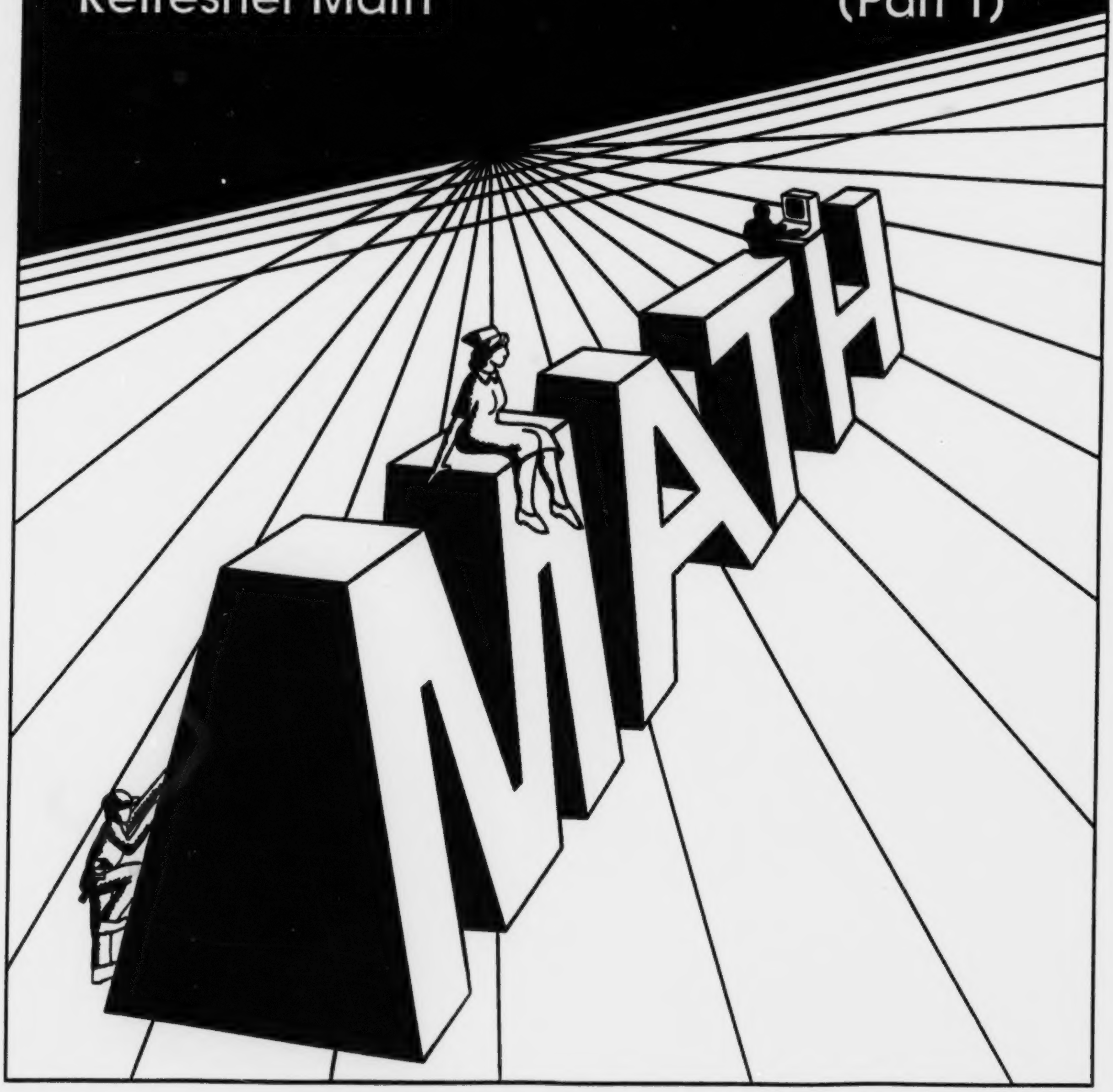
Divide and simplify items 26 and 27.

26. Divide: $1 \div \frac{1}{2} =$

27. Divide: $\frac{3}{4} \div \frac{1}{5} =$

Module 1

Refresher Math (Part 1)



MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Adding and Subtracting Decimals: Lesson 1.1

Even though the focus of this lesson is adding and subtracting decimals, it is suggested that you begin with a brief review of addition and subtraction using whole numbers.

Vocabulary

As a brief introduction to the lesson, review the vocabulary list in the learner text. You may do this as a group discussion, asking for volunteers to state what they think the meaning or significance of each term is, or you may have each learner write his or her own definition for each word. As you go over the correct definition for each term, you may begin to introduce the material that will be covered.

Develop Your Understanding

"Walk through" this section with the learners. You may then proceed into an introduction of the place value for digits in a decimal number followed by a discussion. Use the graph to illustrate place value.

Practice reading decimal numbers. Then discuss how to compare decimal number values to determine which is greater. Review the rules for rounding off whole numbers, and explain that they are the same for decimals. Move into a discussion of the addition of decimals. Point out and illustrate the critical rule of lining up the decimal. Illustrate that one may "carry" across the decimal to higher place value columns.

After explaining addition, make a transition into the subtraction of decimals. Illustrate how to borrow and create 0 digits when there are none. For example, 2 – 0.45 would necessitate writing the 2 as 2.00 and borrowing from the 2 in order to subtract 0.45.

After going through the examples with the learners and discussing the important points to each concept, allow time for questions.

Skill Builders

Learners work independently on this section. Have the learners check their work by using the Answer Key in the back of the book.

Write on Board
When adding and subtracting decimals, line the decimal points up, and add necessary zeros.

Check Yourself

Adults associate decimals with money. As an added activity, you could use play money and give each learner a certain amount to add or subtract.

Lesson 1.1
Adding and Subtracting Decimals

Learning Objectives

To master the basic vocabulary of decimals
To read and compare decimal values
To practice rounding off decimal values
To add and subtract decimals

Decimals are used every day in the workplace. For example, every time you use amounts of money, you are using decimals. Therefore, it is important to understand what decimals mean and know how to use them. This lesson will review what a number with a decimal means, what the places to the right of a decimal point stand for, and how to add and subtract numbers that have decimals.

Can you think of examples of how you use decimals at work or in daily life?

Vocabulary

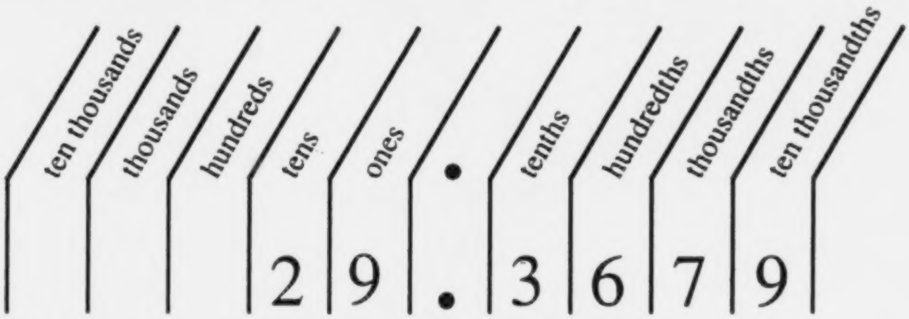
decimal—a number that includes a decimal point and place values to the right of the decimal point (5.2, 65.75, etc.)
tenths—one place value to the right of a decimal point
hundredths—two place values to the right of a decimal point
thousandths—three place values to the right of a decimal point
ten thousandths—four place values to the right of a decimal point

Develop Your Understanding

Decimal Values

What is the value of each number in 29.3679?

The 2 is in the tens place.	2 × 10	= 20
The 9 is in the ones place.	9 × 1	= 9
The 3 is in the tenths place.	3 × 0.1	= 0.3
The 6 is in the hundredths place.	6 × 0.01	= 0.06
The 7 is in the thousandths place.	7 × 0.001	= 0.007
The 9 is in the ten thousandths place.	9 × 0.0001	= 0.0009



How would you read the number aloud: 29.3679?

Read the 29 as you would read a regular whole number. Next, say the word *and* to show the presence of a decimal point. Then read the numbers to the right of the decimal point as if they were a whole number ending with the value place name of the last digit.

For example, 29.3679 would be read: twenty-nine *and* three thousand six hundred seventy-nine ten thousandths. It ends with the words *ten thousandths* because the final number is in the ten thousandths place.

Which is greater, 0.074 or 0.74?

To determine which decimal is greater, line up the decimal points and compare the place values from left to right as you would for whole numbers.

↓
0.074
0.74

The 7 is greater than the 0, so 0.74 is greater than 0.074.

Which is greater, 12.367 or 12.379?

Line up the numbers:

$$\begin{array}{l} \downarrow \\ 12.367 \\ 12.379 \end{array}$$

Compare the two numbers, moving from left to right, until you find a difference:

$$\begin{array}{l} 12.3\underline{6}7 \\ 12.3\underline{7}9 \end{array}$$

There is no difference until the hundredths place. But since 7 is larger than 6, the number 12.379 is greater than the number 12.367.

Once you have found a difference in values, there is no need to compare values that are farther to the right.

Now Try These

a. What is the value of the "6" in the number 5.162? ***six hundredths***

b. Circle the larger number: 0.396 or [✓ 0.485?]

Rounding

How would you round off the number 15.57?

Whenever you round numbers, you must first decide how far to go; you may want to round off to the nearest hundredth, tenth, or to the nearest whole number.

Rules for Rounding Off Numbers:

1. Look one place to the right of your chosen place value.
2. For values of 4 or below, round *down* to 0. Drop all digits to the right.
3. For values of 5 or above, add 1 to the next place value to the left. Drop all digits to the right.

* Remember that an original number is more accurate than a rounded number. Do not round off numbers when exact precision is important!

Example 1

To round 15.57 to the nearest *tenth*, look at the hundredths place. Since 7 is between 5 and 9, round up and add a 1 to the value in the tenths place:

15.57 → 15.6

Example 2

To round 14.294 to the nearest hundredth, look at the thousandths place. Since 4 is less than 5, round this value down to 0:

14.294 → 14.290 → 14.29

When the farthest value to the right of the decimal is a zero, that value can be dropped.

Now Try These

a. Round off 9.212 to the nearest *tenth*. ***9.2***

b. Round off 30.49 to the nearest *whole number*. ***30***

c. Round off 2.106 to the nearest *hundredth*. ***2.11***

Adding and Subtracting

How would you add the following?

$$2.679 + 0.384 = ?$$
$$24.3774 + 23.699 = ?$$

When adding decimals, be sure that you line up the decimal points so that digits from the same value place are being added.

$$\begin{array}{r} 2.679 \\ \underline{+\ .384} \\ 3.063 \end{array} \qquad \begin{array}{r} 24.3774 \\ \underline{+\ 23.699\ } \\ 48.0764 \end{array}$$

The process of adding decimals is exactly the same as the process of adding whole numbers. Be sure that you put the decimal point in the correct place in the solution.

How would you subtract the following?

$1.322 - 0.977 = ?$

$2 - 0.378 = ?$

Remember to line up the decimal points. Notice how a whole number can be written when using it with decimals.

$$\begin{array}{r} 1.322 \\ \underline{-\ 0.977} \\ 0.345 \end{array} \qquad \begin{array}{r} 2.000 \\ \underline{-\ 0.378} \\ 1.622 \end{array}$$

The process of subtracting decimals is also the same as the process of subtracting whole numbers. You may borrow from place values across the decimal point. Be sure that a decimal point is included in the correct place in the solution.

Now Try These

a. 12.494 + 0.36 = ***12.854***

b. 5.573 - 0.689 = ***4.884***

Skill Builders

Read each of these numbers aloud:

1. 14.3 ***Fourteen and three tenths***
2. 15.67 ***Fifteen and sixty-seven hundredths***
3. 22.9984 ***Twenty-two and nine thousand nine hundred eighty-four ten thousandths***

Circle the greater number in each pair:

4. 15 or [✓ 15.2]
5. 2.9 or [✓ 2.93]
6. [✓ 3.678] or 3.54
7. [✓ 1.5] or 1.4

Round these decimals off to the nearest whole number:

8. 15.5 ***16***
9. 16.25 ***16***
10. 3.4 ***3***
11. 0.2 ***0***
12. 4.9 ***5***
13. 1.489 ***1***

Add:

14. 3 + 0.27 + 19.4 = ***22.67***

15. 12.79 + 15.97 = ***28.76***

16. 1.36 + 12.32 + 0.9 = ***14.58***

17. 4.4 + 1.13 + 0.9987 = ***6.5287***

Subtract:

18. $$\begin{array}{r} 49.5 \\ \underline{-\ 3.2} \\ \mathbf{\mathit{46.3}} \end{array}$$

19. $$\begin{array}{r} 50 \\ \underline{-\ \ \ 0.989} \\ \mathbf{\mathit{49.011}} \end{array}$$

20. $$\begin{array}{r} 50.058 \\ \underline{-\ 43.264} \\ \mathbf{\mathit{6.794}} \end{array}$$

21. $$\begin{array}{r} 19 \\ \underline{-\ 7.83} \\ \mathbf{\mathit{11.17}} \end{array}$$

☑

Check Yourself

1. Read 49.3678 aloud. ***Forty-nine and three thousand six hundred seventy-eight ten thousandths***

2. Round off to the nearest tenth: 13.499 ***13.5***

3. Circle the greater number: [✓ 17.864] or 17.834

4. Add: 0.49 + 12 + 174.9832 = ***187.4732***

5. Subtract: 17.008 – 14.679 = ***2.329***

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Multiplying Decimals: Lesson 1.2

Before you begin this lesson, review the multiplication table with learners. A good way to review is for the instructor to demonstrate how the multiplication table is built. Stress the importance of **mastering** the multiplication tables.

Vocabulary

Review the definition of *product.*

Develop Your Understanding

Allow learners to read this section silently. Afterwards, review all steps for multiplying decimals. Paraphrase this section.

Write on Board

Multiply the decimals as you would whole numbers, disregarding the decimal points until you have obtained the product. Then count the number of decimal places involving both factors, and that will be the number of decimal places in the product.

Continue practicing how to read decimals after doing Now Try These.

The examples given attempt to explain place value and derivation of the algorithm used to find the place value of a decimal product. Several examples may be needed before the learner feels comfortable with "counting the number of decimal places."

Skill Builders

The learner works independently on this section. Have the learners check their work in the Answer Key in the back of the book. This method gives the learner immediate feedback.

Check Yourself

Learner works independently as instructor monitors. Allow time to check work as a group.

Closure

Continue review of multiplication by playing the game "Multo." This game can be purchased from any educational bookstore. It is like "Bingo," and adults really find this a **fun** way to review!

Lesson 1.2

Multiplying Decimals

Learning Objectives

To learn to multiply decimals

In this lesson you will have a chance to practice multiplying decimals. Multiplication of decimals is something you may use daily as you figure your income and expenses involving dollars and cents, or if you use measurements at work. Dollars and cents are designated by decimal points, as are measurements like inches, gallons, liters, and centimeters.

Vocabulary

product—the answer to a multiplication problem

$$\begin{array}{r} 12 \\ \underline{\times\ 6} \\ 72 \end{array} \leftarrow \text{product}$$

Develop Your Understanding

The process of multiplying decimals is the same as the process for multiplying whole numbers. However, the product of any multiplication problem with decimals must have the same number of decimal places as the sum of decimal places in the problem.

Steps for Multiplying Decimals

1. Multiply as you usually do with whole numbers. Disregard the decimal point until you have found the product.

2. Count up all the decimal places in the original 2 (or more) numbers.
3. Beginning with the far right place value in the product, count this number of places to the left, and place the decimal point there.

Example 1

Multiply 3.4 by 1.2.

Write the problem vertically:

$$\begin{array}{r} 3.4 \\ \times\ 1.2 \\ \hline \end{array}$$

Multiply as whole numbers, disregarding the decimal points:

$$\begin{array}{r} 34 \\ \times\ 12 \\ \hline 68 \\ 34 \\ \hline 408 \end{array}$$

The number of decimal places in the product must match the total number of places in the original numbers. Start on the right and count left:

$$\begin{array}{rl} 3.4 & \quad 1 \text{ decimal place} \\ \times\ 1.2 & \quad +\underline{1 \text{ decimal place}} \\ \hline 68 & \\ 34 & \\ \hline 4.08 & \quad 2 \text{ decimal places in the product} \end{array}$$

move 2 places left from here

place decimal here

Now Try These

a. $\begin{array}{r} 3.2 \\ \times\ 0.1 \\ \hline \mathbf{0.32} \end{array}$

b. $\begin{array}{r} 2.5 \\ \times\ 0.75 \\ \hline \mathbf{1.875} \end{array}$

Example 2

Do not drop the final zero until *after* the decimal point is placed:

5.08	2 decimal places
× 2.5	+1 decimal place
12.700	3 decimal places in the product

The number "12.700" may then be written "12.7." Place values of zero on the right of the decimal point may be dropped when there are no other numbers farther to the right.

Now Try These

Calculate the products, then rewrite each product, dropping zeros where possible:

a. 3.6
× 3.5
12.6

b. 2.25
× 1.04
2.34

Example 3

A zero can act as a place holder when there are fewer numbers in the product than decimal values in the original numbers:

0.2	1 decimal place
× 0.3	+1 decimal place
0.06	2 decimal places in the product

When you write a decimal without a whole number, you may choose to use a zero as a place holder for the whole number. However, it is not necessary that you do so:

0.06 = .06

Now Try These

a. 3.68
× .02
0.0736

b. .045
× .01
0.00045

Skill Builders

Multiply:

1. 17.1
× 4.9
83.79

2. 6.01
× .07
0.4207

3. 11.5
× 2.4
27.6

4. 0.002
× 5
0.01

Multiply and round each product to the nearest *hundredth* if possible.

5. 3.6
× 25
90

6. 71.28
× .18
12.8304; 12.83 rounded

7. $\begin{array}{r} 3.765 \\ \underline{\times\ 9} \end{array}$

33.885; 33.89 rounded

8. $\begin{array}{r} 45.01 \\ \underline{\times\ 4.09} \end{array}$

184.0909; 184.09 rounded

9. $\begin{array}{r} 437.3 \\ \underline{\times\ 22} \end{array}$

9,620.6

10. $\begin{array}{r} 16 \\ \underline{\times\ 41.56} \end{array}$

664.96

☑ Check Yourself

Use your *multiplication* skills to solve these problems.

1. If 6 people each contribute $4.25 to a birthday party fund, how much money will be in the fund?

   $25.50

2. Larry works 7.5 hours every day from Monday through Friday. How many hours does he work in a week?

   37.5 hours

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Dividing with Decimals: Lesson 1.3

To put focus on this lesson, ask a leading question. What is the opposite of multiplication? Review the multiplication table, and show how to divide using the table.

Vocabulary

Write the words on the board, and discuss definitions with learners. You may compare the divisor to a knife and the dividend to something to be cut. Review the meaning of *product* (answer to a multiplication problem), and contrast with *quotient* (answer to a division problem).

Develop Your Understanding

Show what is meant by moving "the decimal point to the right until it becomes a whole number." Emphasize the importance of moving the decimal an equal number of times for dividend and divisor. Insist that learners insert a decimal point in the quotient before solving. Watch for a common mistake: Some learners routinely insert a zero after the decimal point in the quotient because they forget to "drop down" zeros to the right of the decimal point. You may want to assign only a few problems at a time, then go over with the class. Some learners need extra help with the concept of dividing a number by a greater number in Example 3. Work through one example at a time, and have learners try the problems in the Now Try These sections before moving to the next example.

Skill Builders

Learners work independently on these problems. Have the learners check their work by using the Answer Key in the back of the book.

Check Yourself

Learners with limited reading skills may need help setting up the problems.

Closure

Ask learners for more specific examples of occasions when the skills taught in this lesson will be useful, or have them create their own word problems for the class to solve. Problem solving could be done in teams, with one team creating a problem to solve and passing it to another team to solve; of course, the first team must provide a problem that they themselves have solved!

If time allows, practice multiplying and dividing decimals by powers of 10.

Tip: When *multiplying* by powers of 10, move the decimal point *to the right* the same number of places as there are zeros in the power of 10. When *dividing* by powers of 10, move the decimal point *to the left* the same number of places as there are zeros in the divisor.

Lesson 1.3
Dividing with Decimals

Learning Objectives

To practice dividing combinations of decimals and whole numbers
To learn to divide numbers by larger numbers

The ability to divide numbers that include decimals is an important skill both at work and outside of work. At work, you may need to divide something into equal parts or use very exact measurements. Outside of work, you may need to divide a bill equally between several people or calculate how much money you can afford to spend each day. In this lesson, you will learn division using different combinations of whole numbers and decimals. You will also learn how to divide a number by a larger number, an important step toward learning to calculate percentages.

Vocabulary

dividend—a number to be divided
divisor—a number to divide the dividend by
quotient—the answer to a division problem

$$\begin{array}{rrl} & 5 & \leftarrow \textit{quotient} \\ \textit{divisor} \rightarrow 6 & \overline{)30} & \leftarrow \textit{dividend} \\ & \underline{-30} & \\ & 0 & \end{array}$$

Develop Your Understanding

All division problems have a dividend and a divisor. The dividend is the number that will be "cut into pieces"; the divisor tells you how many pieces to cut.

In horizontal form, the dividend comes first:

$$\underset{\text{dividend}}{12} \div \underset{\text{divisor}}{3} = \underset{\text{quotient}}{4}$$

In vertical form, the dividend is on the inside of the division symbol:

$$\begin{array}{r} 4 \leftarrow \text{quotient} \\ \text{divisor} \rightarrow 3\overline{)12} \leftarrow \text{dividend} \end{array}$$

Steps for Dividing with Decimals

1. If the divisor has a decimal point in it, move the decimal point to the right until it becomes a whole number.
2. Move the decimal point in the dividend to the right an equal number of places.
3. Insert the decimal point in the place where the quotient will go.
4. Perform division.

Changing the Divisor

Division should not be performed while the divisor has a decimal point in it. When the divisor has a decimal point, move the decimal point to the right as many places as necessary to make the divisor a whole number. Next, move the decimal point in the dividend to the right an equal number of places. YOU MUST change the dividend and divisor in exactly the same way. It is all right if the dividend still contains a decimal.

Example 1

$$18.2\overline{)36.4}$$

Move the decimal point in the divisor one place to the right. This changes 18.2 to 182:

$$18.\underset{\smile}{2} \rightarrow 182.$$

Now, move the decimal point in the dividend one place to the right. This changes 36.4 to 364:

$$36.\underset{\smile}{4} \rightarrow 364.$$

The problem looks like this:

$$\begin{array}{r} 2. \\ 182\,\overline{)364.} \\ \underline{-364} \\ 0 \end{array}$$

The quotient is 2.

Now Try These

a. $\overset{3}{2.56\,\overline{)7.68}}$ b. $\overset{4}{2.04\,\overline{)8.16}}$

Decimals in the Quotient

Example 2

$$0.25\,\overline{)31.6}$$

In this case, the divisor contains two decimal places, but the dividend contains only one.

> *Adding zeros to the far right of a decimal does not change the decimal's value. For example, 1.25 = 1.25000.*

Move the decimal point two places over in both the divisor and the dividend:

$$0.\underset{\smile\smile}{25} \rightarrow 25. \qquad 31.\underset{\smile\smile}{6\;\,} \rightarrow 3{,}160.$$

Add zeros on the right as place holders when you move the decimal point more times than there are place values.

Before you start to divide, put a decimal point directly above the decimal point in the dividend, in the place where you will fill in the quotient:

$$\begin{array}{r} . \\ 25\,\overline{)3160.} \end{array}$$

Begin dividing. Once you are on the right side of the decimal point, you can continue to add zeros to the right of the remainder when needed.

$$\begin{array}{r} 126.4 \\ 25\,\overline{)3160.} \\ \underline{-25} \\ 66 \\ \underline{-50} \\ 160 \\ \underline{-150} \\ 100 \\ \underline{-100} \\ 0 \end{array}$$

Now Try These

Round to the nearest *thousandth.*

a. $31\,\overline{)622}$ = ***20.065***

b. $19\,\overline{)42}$ = ***2.211***

Example 3:

Using decimals, you can divide a number by a greater number.

$$\begin{array}{r} 0 \\ 130\,\overline{)65} \end{array}$$

When the divisor is greater than the dividend, begin by inserting a decimal point into the quotient. You may then add a zero to the dividend.

$$\begin{array}{r} 0. \\ 130\,\overline{)65.0} \end{array}$$

Next, proceed to divide as usual, inserting the quotient to the right of the decimal point.

$$\begin{array}{r} 0.5 \\ 130\,\overline{)\,65.0} \\ \underline{-650} \\ 0 \end{array}$$

The quotient is 0.5.

Now Try These

a. $12\,\overline{)\,3}$ = ***0.25***

b. $6\,\overline{)\,2.4}$ = ***0.4***

Skill Builders

Calculate the following problems. Round all quotients off to *two* decimal places.

1. $2\,\overline{)\,8.24}$ = ***4.12***

2. $27\,\overline{)\,168.75}$ = ***6.25***

3. $2.5\,\overline{)\,7.5}$ = ***3***

4. $3.62\,\overline{)\,5.068}$ = ***1.4***

5. $1.008\overline{)6}$ = ***5.95***

6. $51\overline{)143}$ = ***2.80 or 2.8***

7. $3\overline{)56}$ = ***18.67***

8. $0.03\overline{)162}$ = ***5,400***

9. $42\overline{)21}$ = ***0.5***

10. $250\overline{)10}$ = ***0.04***

Check Yourself

Use your *division* skills to solve these problems.

1. The parts department received an order for a part that costs $1.47 each. The customer forgot to fill in how many parts she wanted on the order form, but enclosed a check for $5.88. How many parts will this pay for?

   4 parts

2. Eleven people on a manufacturing line won a $1,000 reward for a cost-saving suggestion and split the money equally. About how much money did each person get?

   $90.91

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Using Fractions: Adding, Subtracting, and Reducing to Lowest Terms: Lesson 1.4

Even though this lesson provides instruction on adding and subtracting fractions with like denominators, certain skills must be mastered.

Throughout the lesson, the icon ✎ will designate a time to "break," checking for mastery. When the icon appears in a Now Try These section, it is suggested that learners work the Skill Builders section that correlates with the concept being taught.

Vocabulary

Use a chalkboard or overhead to review the definitions.

Develop Your Understanding

Read examples and continue guided discussion of mixed numbers, improper fractions, and proper fractions. To be sure that students understand the definitions of improper fractions and mixed numbers, have them give additional examples of each. Converting a mixed number to an improper fraction and an improper fraction to a mixed number is a very important *prerequisite* skill for doing all operations with fractions.

Adding Fractions with the Same Denominator

Instruct learners to note that, when adding fractions with the same denominator, you add the numerators, and the denominator remains the same.

Reducing Fractions to Their Lowest Terms

You may want to introduce this section by showing how difficult it is to compare fractions that are not reduced.

Review the concept of the greatest common factor, and demonstrate finding the GCF. You may also show that fractions can be reduced in several steps instead of just one—that is, the factors of the GCF. Tricks of the trade include:

- All even numbers can be reduced by 2.
- All numbers ending in 5 or 0 can be reduced by 5.
- All numbers ending in 0 can be reduced by 10.

Tip: To reduce a fraction to lowest terms, ask the question: What is the *largest* whole number that will divide both the numerator and denominator of this fraction?

Skill Builders

Learners work independently on this section. Have learners check their work in the Answer Key.

Check Yourself

Learner works independently as you monitor. Allow time to check work as a group.

Closure

Learner summarizes skills by playing "Rolling Fractions with No Wholes." This game provides fun, drill, higher-level thinking, and cooperative approaches to learning. The focus of this game is to *reinforce* recognition of proper and improper fractions.

Players—class and instructor
Materials—4 cubes marked 0-9

Give two cubes to the class, and keep two cubes for yourself. Each team rolls one cube for the numerator and one for the denominator. The team that rolls 5 proper or 5 improper fractions is the winner. If a whole number like $\frac{5}{5}$ is rolled, the team must start over.

Lesson 1.4

Using Fractions: Adding, Subtracting, and Reducing to Lowest Terms

Learning Objectives

To develop an understanding of fractional terms and their meanings
To recognize different types of fractions
To practice adding and subtracting fractions that have the same denominator
To practice reducing fractions to their lowest terms

The ability to understand and use fractions is becoming more important as modern workplaces shift to emphasize technical skills. You will need to review the vocabulary of fractions, and to quickly recognize the three types of fractions. In this lesson, you will learn to change improper fractions to mixed numbers and mixed numbers to improper fractions. You will also practice adding and subtracting fractions with the same denominator, and learn to reduce fractions to lowest terms.

Give three examples of how fractions are used in your job or in other jobs in your area.

1.____
2.____
3.____

Vocabulary

fraction—part of a whole. In math, this is expressed in the form: $\frac{\text{part}}{\text{whole}}$
bar—the short line separating a numerator from a denominator; also a symbol for division
denominator—the bottom number in a fraction
numerator—the top number in a fraction

terms—the numerator and denominator comprise the terms of a fraction
proper fraction—a fraction with a smaller numerator than denominator
improper fraction—a fraction with a numerator equal to or greater than the denominator
mixed number—the sum of a whole number and a proper fraction
greatest common factor (GCF)—the largest number that will divide evenly into both the numerator and denominator of a fraction

---

Develop Your Understanding

Types of Fractions

INSTRUCTIONS

To prepare a bath for plating:

1. Start with $\frac{1}{2}$ gallon of water at 60° to 80°F.
2. Add $\frac{1}{8}$ gallon of solution A.
3. With rapid agitation, add $\frac{3}{16}$ gallon of solution B.
4. Add water to bring volume to $1\frac{1}{2}$ gallons.
5. Add five $\frac{1}{4}$-gallon portions of solution C, or $\frac{5}{4}$ gallons.

There are three types of fractions in this work order. They may be classified as follows:

1. $\frac{1}{2}$ is a proper fraction
2. $\frac{1}{8}$ is a proper fraction
3. $\frac{3}{16}$ is a proper fraction
4. $1\frac{1}{2}$ is a mixed number
5. $\frac{5}{4}$ is an improper fraction

A *proper fraction* shows a quantity less than 1. The numerator of a proper fraction is always smaller than the denominator.

An *improper fraction* shows a quantity equal to or greater than 1. The numerator of an improper fraction is the same as or greater than the denominator.

A *mixed number* is another way to write a fraction that is greater than 1. A mixed number is a whole number plus a proper fraction.

Changing Improper Fractions and Mixed Numbers

Steps for changing an improper fraction to a mixed number:

1. Determine how many times the denominator will divide evenly into the numerator. This will be the whole number part.
2. Subtract the amount taken out for the whole number from the numerator. The remainder will be the new numerator.
3. Keep the same denominator.

Example 1

For the improper fraction $\frac{5}{4}$:

Step 1: 4 goes into 5 one time, so the whole number will be 1.
Step 2: After subtracting 4 from the numerator, the remainder is 1, so the numerator will be 1.
Step 3: The denominator will still be 4.

$\frac{5}{4}$ $5 \div 4 = 1$ with remainder of 1 $\frac{5}{4} = 1\frac{1}{4}$

Steps for changing a mixed number to an improper fraction:

1. Multiply the denominator of the fraction by the whole number.
2. Add the numerator of the fraction to the product.
3. Write the result over the original denominator.

Example 2

For the mixed number $1\frac{5}{8}$:

Step 1: The product of the whole number and the denominator is 8.
Step 2: 8 plus the numerator (5) is 13. This is the new numerator.
Step 3: The denominator will still be 8.

$1\frac{5}{8}$ $8 \times 1 = 8$ $8 + 5 = 13$ $1\frac{5}{8} = \frac{13}{8}$

Now Try These

a. Change $\frac{8}{5}$ to a mixed number. ________________

$\mathbf{1\frac{3}{5}}$

b. Change $5\frac{1}{2}$ to an improper fraction. ________________

$\mathbf{\frac{11}{2}}$

Adding Fractions

When fractions have the same denominator, add only the numerators, leaving the denominators as they are.

Example 3

$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

Now Try These

a. $\frac{2}{7} + \frac{3}{7} =$ ________________________ $\mathbf{\frac{5}{7}}$

b. $\frac{1}{15} + \frac{7}{15} =$ ________________________ $\mathbf{\frac{8}{15}}$

Subtracting Fractions

When subtracting fractions with the same denominator, subtract only the numerators and leave the denominators as they are.

Example 4

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$

Now Try These

a. $\frac{6}{7} - \frac{1}{7} =$ ____________________ $\mathbf{\frac{5}{7}}$

b. $\frac{2}{3} - \frac{1}{3} =$ ____________________ $\mathbf{\frac{1}{3}}$

Reducing Fractions to Their Lowest Terms

When working with fractions, it is important to simplify them or, in other words, reduce them to their lowest terms. To do this you need to find the *greatest common factor* (GCF) for the numerator and the denominator. This is the greatest number that will divide into both the numerator and the denominator. When working with an improper fraction, remember to reduce it by making it a mixed number and then reducing the remainder by the GCF.

Example 5

$\frac{40}{60}$ will reduce to $\frac{2}{3}$ by dividing both the numerator and the denominator by 20, which is the greatest common factor.

$40 \div 20 = 2$

$$\frac{40}{60} = \frac{2}{3}$$

$60 \div 20 = 3$

Example 6

$\frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

$2 \div 2 = 1$

$4 \div 2 = 2$

$\frac{2}{4} = \frac{1}{2}$

$\frac{6}{4}$ is reduced by making it a mixed number and then reducing the remainder by dividing it by 2 (the GCF). The whole number does not change when the fraction is reduced to lowest terms.

Now Try These

Reduce each fraction to lowest terms, using the GCF.

a. $\frac{4}{8}$ = ____________________ $\frac{1}{2}$

b. $\frac{10}{15}$ = ____________________ $\frac{2}{3}$

c. $\frac{9}{3}$ = ____________________ *3*

Skill Builders

Circle the proper fraction in each pair.

1. [✓ $\frac{1}{2}$] or $\frac{4}{3}$
2. [✓ $\frac{6}{7}$] or $\frac{7}{6}$
3. $\frac{3}{2}$ or [✓ $\frac{5}{7}$]

Change each improper fraction to a mixed numeral.

4. $\frac{7}{3}$ = ________________________ $2\frac{1}{3}$
5. $\frac{16}{5}$ = ________________________ $3\frac{1}{5}$
6. $\frac{18}{7}$ = ________________________ $2\frac{4}{7}$
7. $\frac{40}{9}$ = ________________________ $4\frac{4}{9}$

Change each mixed numeral to an improper fraction.

8. $1\frac{1}{3} =$ ______________________ $\mathbf{\frac{4}{3}}$

9. $6\frac{7}{8} =$ ______________________ $\mathbf{\frac{55}{8}}$

10. $8\frac{1}{4} =$ ______________________ $\mathbf{\frac{33}{4}}$

11. $13\frac{2}{3} =$ ______________________ $\mathbf{\frac{41}{3}}$

Add the fractions.

12. $\frac{1}{5} + \frac{2}{5} =$ ________________ $\mathbf{\frac{3}{5}}$

13. $\frac{2}{3} + \frac{2}{3} =$ ________________ $\mathbf{\frac{4}{3} = 1\frac{1}{3}}$

14. $\frac{7}{8} + \frac{2}{8} =$ ________________ $\mathbf{\frac{9}{8} = 1\frac{1}{8}}$

15. $\frac{11}{12} + \frac{1}{12} =$ ______________ $\mathbf{\frac{12}{12} = 1}$

Subtract the fractions.

16. $\frac{3}{6} - \frac{2}{6} =$ ________________ $\mathbf{\frac{1}{6}}$

17. $\frac{7}{13} - \frac{3}{13} =$ ______________ $\mathbf{\frac{4}{13}}$

18. $\frac{8}{10} - \frac{5}{10} =$ ______________ $\mathbf{\frac{3}{10}}$

19. $\frac{8}{15} - \frac{6}{15} =$ ______________ $\mathbf{\frac{2}{15}}$

Reduce each fraction to lowest terms.

20. $\frac{8}{10} =$ ______ **$\frac{4}{5}$**

21. $\frac{30}{60} =$ ______ **$\frac{1}{2}$**

22. $\frac{3}{12} =$ ______ **$\frac{1}{4}$**

23. $\frac{16}{12} =$ ______ **$1\frac{1}{3}$**

24. $\frac{20}{15} =$ ______ **$1\frac{1}{3}$**

☑

Check Yourself

Problem: A new mix of chemicals has been developed that will improve the quality of the plating operation. Suppose that you are mixing it by hand, using this formula:

$\frac{1}{8}$ gallon of substance A

$\frac{3}{8}$ gallon of substance B

$\frac{6}{8}$ gallon of substance C

$\frac{5}{8}$ gallon of substance D

1. If you mix the chemicals and check the total measurement, what is the total amount of the mixture you have so far? ______

$\frac{15}{8}$ or $1\frac{7}{8}$ gallons

2. You need to pour off some of the mixture so that you will only have $1\frac{1}{2}$ gallons. How much should you pour off? ______

$\frac{3}{8}$ gallon

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Finding Common Denominators: Lesson 1.5

This lesson provides instruction and practice for learners on renaming fractions using their common and/or least common denominators. Relate learner's work experience to the learning objectives by reviewing the previous lesson. You may want to allow two class periods for this topic.

Vocabulary

Review vocabulary from previous lesson.

Develop Your Understanding

Read examples and work them on the overhead or board. Check learner comprehension during this guided practice. The relationship between least common denominators and common denominators should be discussed if it is unclear to learners. It may be helpful to point out that one can always find a common denominator by taking the product of all the given denominators, but this may not be the least common denominator. Remind learners that renaming fractions using the least common denominator is a *prerequisite* skill for adding and subtracting fractions.

Skill Builders

Learners work independently on this activity with your supervision and assistance if needed. Peer assistance may be encouraged during this time. Have learners check their work by using the Answer Key in the back of the book.

Check Yourself

Suggestion: Divide the class into two teams. The first team to solve both problems wins.

Closure

If time allows, continue playing in teams. Make up fractions, and have each team find the least common denominator.

Lesson 1.5
Finding Common Denominators

Learning Objectives

To learn to add and subtract fractions with like and unlike denominators, and mixed numbers

In Lesson 1.4, you reviewed the basics of fractions and practiced adding and subtracting fractions with the same denominator. However, it is often necessary to add or subtract fractions with different denominators. For example, when adding hours worked on a time sheet, you may need to add fractions of an hour such as $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{5}$. You must convert such fractions to fractions with the same denominator before adding or subtracting.

Vocabulary

multiple—the product of a whole number and another whole number
least common multiple (LCM)—the lowest number that two or more numbers can be multiplied into evenly
least common denominator (LCD)—the LCM of two or more denominators
equivalent fractions—fractions that may look different but are equal in value

Develop Your Understanding

Adding and Subtracting Fractions with Unlike Denominators

In order to add or subtract fractions with unlike denominators, they must first be converted to fractions with the same denominators.

Steps for Adding or Subtracting Fractions with Unlike Denominators:

1. Find the least common multiple (LCM) of the denominators. This number is the least common denominator (LCD).
2. Convert each fraction to an equivalent fraction, each having the least common denominator (LCD).
3. Add or subtract the numerators only, keeping the same denominator.

Step 1: Finding the Least Common Multiple

The *multiples* of a number are the products of that number and other whole numbers, including 1.

Example 1

The first 5 multiples of the number 2 are 2, 4, 6, 8, and 10:

$$\begin{aligned} 2 \times 1 &= 2 \\ 2 \times 2 &= 4 \\ 2 \times 3 &= 6 \\ 2 \times 4 &= 8 \\ 2 \times 5 &= 10 \end{aligned}$$

Now Try These

List the first 5 multiples of each number:

a. 3: *3*, *6*, *9*, *12*, *15*.

b. 7: *7*, *14*, *21*, *28*, *35*.

When you have two or more fractions with unlike denominators to add or subtract, begin by comparing the denominators and listing multiples of each denominator. The smallest number that is a multiple of all the denominators is called the *least common multiple (LCM).*

Example 2

$$\frac{1}{3} + \frac{1}{4}$$

Find the LCM of 3 and 4.

Multiples of 3 are: 3, 6, 9, 12, 15, 18.......

Multiples of 4 are: 4, 8, 12, 16, 20, 24.....

12 is the LCM of 3 and 4.

Sometimes one of the denominators will also be the LCM.

Example 3

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

Find the LCM of 2, 4, and 8.

Multiples of 2 are: 2, 4, 6, 8, 10, 12...

Multiples of 4 are: 4, 8, 12, 16, 20, 24...

Multiples of 8 are: 8, 16, 24, 32, 40, 48...

8 is the LCM of 2, 4, and 8.

Now Try These

Find the LCM of each of the following:

a. 5 and 10 ___________

10

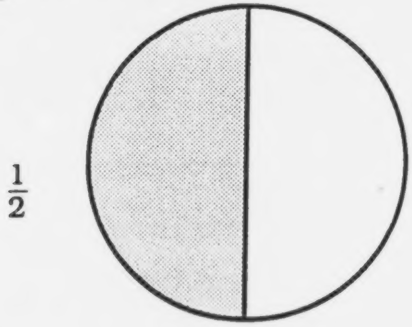
b. 3, 6, and 8 __________

24

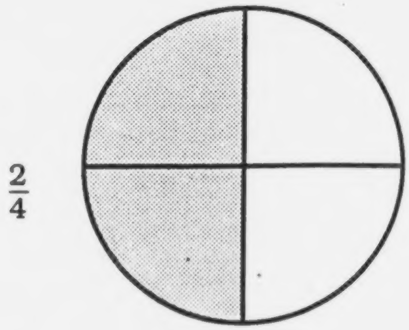
Step 2: Converting to Equivalent Fractions

Fractions that have the same value are sometimes called by different names. Fractions that look different but are equal in value are called *equivalent fractions*.

Circle A is divided into 2 parts. The shaded portion of Circle A is $\frac{1}{2}$ of the circle. Circle B, on the other hand, is divided into 4 parts. The shaded portion of Circle B is the same size as the shaded portion of Circle A. It is also $\frac{2}{4}$ of the circle.



Circle A



Circle B

Since $\frac{1}{2}$ of a circle is equivalent to $\frac{2}{4}$ of the same circle, $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions:

$$\frac{1}{2} = \frac{2}{4}$$

You can compute equivalent fractions without using circles.

> *If you multiply both the numerator and the denominator of any fraction by the same number, the result will be a fraction equivalent to the original one.*

The above rule is true because any fraction that has the same numerator and denominator is equal to 1; and any number multiplied by 1 is equal to itself.

Example 4

Find 2 fractions that are equivalent to $\frac{2}{3}$.

$$\frac{2 \times 2 = 4}{3 \times 2 = 6} \qquad \frac{2 \times 3 = 6}{3 \times 3 = 9}$$

Therefore, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$.

Now Try These

Find two equivalent fractions for each example:

a. $\frac{3}{4}$ ***Answers will vary. Examples are*** $\frac{6}{8}$, $\frac{9}{12}$, ***etc.***

b. $\frac{1}{7}$ ***Answers will vary. Examples are*** $\frac{2}{14}$, $\frac{3}{21}$, ***etc.***

Least Common Denominator

Suppose you want to add two fractions with unlike denominators:

$$\frac{1}{6} + \frac{1}{9} = ?$$

Checking the multiples of 6 and 9, you find that the LCM is 18:

Multiples of 6: 6, 12, 18, 24....

Multiples of 9: 9, 18, 27, 36....

The next step is to convert both fractions to equivalent fractions with the same denominator. The LCM is the number you will use for the denominator. This number is also called the *LCD* or *least common denominator:*

$$\frac{1}{6} = \frac{?}{18} \qquad \frac{1}{9} = \frac{?}{18}$$

To compute equivalent fractions using the LCD, work backwards. Find the number that the original denominator must be multiplied by to result in the LCD. Then, multiply the numerator of the fraction by the same number:

$\frac{1}{6} = \frac{?}{18}$ 6 must be multiplied by **3** to get 18.

$\frac{1 \times 3 = 3}{6 \times 3 = 18}$ Multiply the numerator by **3**, too.

$\frac{1}{6}$ is converted to $\frac{3}{18}$.

Next, repeat the same process for the fraction $\frac{1}{9}$:

$\frac{1}{9} = \frac{?}{18}$ 9 must be multiplied by **2** to get 18.

$\frac{1 \times 2 = 2}{9 \times 2 = 18}$ Multiply the numerator by **2** as well.

$\frac{1}{9}$ is converted to $\frac{2}{18}$.

The problem can now be solved:

$$\frac{1}{6} + \frac{1}{9} = \frac{3}{18} + \frac{2}{18} = \mathbf{\frac{5}{18}}$$

Now Try These

Use this problem to answer each item:

$$\mathbf{\frac{5}{6} + \frac{1}{3} =}$$

a. Find the LCM:

6

b. Convert to equivalent fractions containing the LCD:

$\mathbf{\frac{5}{6} + \frac{2}{6} =}$

c. Add and reduce to lowest terms:

$\mathbf{\frac{7}{6} = 1\frac{1}{6}}$

Working with Mixed Numbers

When adding mixed numbers, group the whole numbers separately from the fractions, add, and combine the results:

Example 5

$$3\frac{1}{2} + 6\frac{1}{4} =$$

$3 + 6 + \frac{1}{2} + \frac{1}{4}$	*Separate whole numbers from fractions.*
$3 + 6 = 9$	*Add the whole numbers.*
$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$	*Add the fractions.*
$9 + \frac{3}{4} = \mathbf{9\frac{3}{4}}$	*Combine the two sums; be sure to put the result in lowest terms.*

Now Try These

a. $12\frac{1}{3} + 4\frac{1}{2} = \mathbf{16\frac{5}{6}}$

b. $2\frac{1}{14} + 6\frac{3}{7} = \mathbf{8\frac{1}{2}}$

When subtracting mixed numbers, you may need to "borrow" a whole number to subtract the fractions:

Example 6

$5\frac{1}{5} - 2\frac{3}{10} =$	*Convert the fractions with unlike denominators to fractions with the same denominator.*
$5\frac{2}{10} - 2\frac{3}{10} =$	*Since you cannot subtract* $\frac{3}{10}$ *from* $\frac{2}{10}$, *borrow* $\frac{10}{10}$ *from the "5."*

$4\frac{12}{10} - 2\frac{3}{10}$ *Separate the whole numbers and fractions.*

$4 - 2 = 2$ *Subtract the whole numbers.*

$\frac{12}{10} - \frac{3}{10} = \frac{9}{10}$ *Subtract the fractions.*

$5\frac{2}{10} - 2\frac{3}{10} = \mathbf{2\frac{9}{10}}$ *Combine the results.*

Now Try These

a. $8\frac{1}{8} - 7\frac{3}{4} = \mathbf{\frac{3}{8}}$

b. $4\frac{3}{5} - 1\frac{1}{2} = \mathbf{3\frac{1}{10}}$

Skill Builders

Add or subtract, and reduce to lowest terms.

1. $\frac{1}{5} + \frac{2}{10} = \mathbf{\frac{2}{5}}$

2. $\frac{1}{3} + \frac{3}{4} = \mathbf{1\frac{1}{12}}$

3. $\frac{2}{4} + \frac{5}{8} = \mathbf{1\frac{1}{8}}$

4. $\frac{2}{3} + \frac{4}{5} = \mathbf{1\frac{7}{15}}$

5. $\frac{5}{6} + \frac{1}{4} = \mathbf{1\,\frac{1}{12}}$

6. $\frac{3}{7} + \frac{1}{4} = \mathbf{\frac{19}{28}}$

7. $\frac{3}{8} - \frac{1}{6} = \mathbf{\frac{5}{24}}$

8. $\frac{5}{8} - \frac{1}{2} = \mathbf{\frac{1}{8}}$

9. $\frac{8}{9} - \frac{5}{6} = \mathbf{\frac{1}{18}}$

10. $\frac{1}{2} - \frac{2}{7} = \mathbf{\frac{3}{14}}$

11. $8\frac{1}{2} + 10\frac{2}{3} = \mathbf{19\,\frac{1}{6}}$

12. $1\frac{3}{8} + 2\frac{3}{4} = \mathbf{4\,\frac{1}{8}}$

13. $9\frac{2}{3} - 4\frac{2}{6} = \mathbf{5\,\frac{1}{3}}$

14. $14\,\frac{7}{10} - 3\,\frac{4}{15} = \mathbf{11\,\frac{13}{30}}$

Check Yourself

Problem 1:

Job Task	Process Time
black oxide	$1\frac{4}{6}$ hr
layup	$\frac{2}{3}$ hr
lamination	$4\frac{1}{2}$ hr

What is the total process time for all job tasks?

Total process time = ________________________ **$6\frac{5}{6}$ hours**

Problem 2:

Job: Cutter
Task: Program cut measurements into machine.

Cut 1: $8\frac{1}{2}$ inches
Cut 2: $\frac{3}{16}$ inch
Cut 3: $\frac{3}{8}$ inch

Total inches cut = ________ **$9\frac{1}{16}$ inches**

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Multiplying and Dividing Fractions: Lesson 1.6

Since the focus of this lesson is multiplying and dividing fractions, a brief review of prerequisite skills is needed.

Vocabulary

Review all vocabulary relating to fractions as well as terms relating to multiplication and division before introducing "reciprocal."

Write fractions on the board, and have learners give their reciprocals. Mixed numbers should be converted to improper fractions before determining reciprocals.

Develop Your Understanding

One approach to this lesson would be to cover all of the information and Skill Builders related to multiplication before moving on to division, although a comprehensive approach is also possible since the two skills are quite similar. Depending on the skill level of the student, the lesson may also be taught in two successive classes. Look for the icon ✎ .

Multiplication skills:	Examples 1-3 under this heading in Develop Your Understanding, plus Skill Builders 1-10.
Division skills:	Examples 4-5 under this heading in Develop Your Understanding, plus Skill Builders 11-20.

Skill Builders

Learners may work individually, in pairs, or in groups. Have learners check their work by using the Answer Key in the back of the book.

Check Yourself

Demonstrate this shortcut several times. Reassure those who are confused by the technique that it is only a shortcut and need not be used to obtain the correct answer.

Closure

If time allows, give learners word problems to help them apply their new skills. A fairly complex example:

> Production rates at ABC Computer Company were unusually high for the first quarter of 1990 at 4,680 computers produced and shipped. During the second quarter, only $\frac{4}{5}$ as many computers were produced, and during the third quarter the company had closed one plant and produced only half as many computers as the total for the second quarter.
>
> How many computers were produced by ABC in the second and third quarters of 1990?
>
> Solution:
>
> 2nd quarter: $4{,}680 \times \frac{4}{5} = \frac{18720}{5} = 3{,}744$ produced
>
> 3rd quarter: $3{,}744 \times \frac{1}{2} = \frac{3744}{2} = 1{,}872$ produced

Lesson 1.6
Multiplying and Dividing Fractions

Learning Objectives

To practice multiplying fractions and mixed numbers
To practice dividing fractions and mixed numbers

Now that you have practiced adding and subtracting fractions with both like and unlike denominators, you can enhance your skill with fractions by learning to multiply and divide them. You may have done this before in the following way:

A pamphlet is $8\frac{1}{2}$ inches long. If you want to make a pamphlet that is half the size of this one, how long would it be?______________________

$4\frac{1}{4}$ inches

Vocabulary

reciprocal—the reciprocal of a fraction is that fraction with its numerator and denominator inverted.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$: $\left(\frac{2}{3}\right)$ inverted = $\frac{3}{2}$

Develop Your Understanding

Multiplying Fractions

If you know how to multiply whole numbers, multiplying fractions is not difficult.

> *To multiply two fractions, multiply the numerators and multiply the denominators.*

Example 1

Suppose you want to find out what $\frac{1}{2}$ of the fraction $\frac{3}{7}$ is. You can set this up as a multiplication problem:

$$\frac{3}{7} \times \frac{1}{2} =$$

To get the answer, multiply the two numerators, and follow the same procedure with the two denominators.

$$\frac{3}{7} \times \frac{1}{2} = \frac{3 \times 1}{7 \times 2} = \mathbf{\frac{3}{14}}$$

The product is $\frac{3}{14}$. We can also say that "$\frac{3}{14}$ is $\frac{3}{7}$ of the fraction $\frac{1}{2}$."

Now Try These

Be sure to reduce all answers to lowest terms if applicable.

a. $\frac{2}{3} \times \frac{4}{5} = \mathbf{\frac{8}{15}}$

b. $\frac{1}{8} \times \frac{1}{8} = \mathbf{\frac{1}{64}}$

Whole Numbers

The product of a fraction and a whole number tells the value of that part of the whole. For example, if you wanted to know what a fifth of the number 3 is, you could multiply $\frac{1}{5}$ by 3. In order to solve the problem, 3 can be rewritten as a fraction.

> *Any whole number can be expressed as a fraction by putting the whole number over a denominator of "1."*

Example 2

To multiply $\frac{1}{5}$ by 3, change 3 to the fraction $\frac{3}{1}$:

$$\frac{1}{5} \times 3 = \frac{1}{5} \times \frac{3}{1} = \frac{1 \times 3}{5 \times 1} = \frac{3}{5}$$

The product means that $\frac{1}{5}$ of the number 3 is $\frac{3}{5}$.

Now Try These

Multiply and reduce answers to lowest terms.

a. $\frac{5}{6} \times 2 = \mathbf{1\frac{2}{3}}$

b. $8 \times \frac{9}{10} = \mathbf{7\frac{1}{5}}$

Mixed Numbers

Convert mixed numbers to improper fractions before you multiply.

Example 3

$$3\frac{1}{2} \times 1\frac{1}{3} = \frac{7}{2} \times \frac{4}{3} = \mathbf{\frac{28}{6} = 4\frac{2}{3}}$$

Now Try These

a. $2\frac{1}{4} \times 1\frac{1}{9} = \mathbf{2\frac{1}{2}}$ b. $1\frac{1}{5} \times 1\frac{3}{8} = \mathbf{1\frac{13}{20}}$

Dividing Fractions

When dividing fractions, you must use the *reciprocal* of the divisor, or second fraction in the problem. To find the reciprocal of a fraction, reverse the numerator and the denominator:

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

The reciprocal of a whole number has a numerator of 1 over a denominator of the whole number: The reciprocal of 3 is $\frac{1}{3}$.

To divide two fractions, ***multiply*** *the first fraction by the reciprocal of the second fraction.*

Example 4

$$\frac{1}{2} \div \frac{1}{4} =$$

The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$:

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$$

You can now solve the problem using multiplication:

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} \quad = \quad \frac{1 \times 4}{2 \times 1} = \frac{4}{2} = 2$$

Notice that when you divide a fraction by a fraction, the quotient may be greater than the number you started out with.

Now Try These

a. $\frac{2}{5} \div \frac{8}{9} = \mathbf{\frac{9}{20}}$

b. $\frac{1}{2} \div \frac{3}{4} = \mathbf{\frac{2}{3}}$

Mixed Numbers

Convert mixed numbers to improper fractions when you divide. Then, take the reciprocal of the second improper fraction and multiply.

Example 5

$$1\frac{5}{6} \div \frac{2}{3} = \frac{11}{6} \div \frac{2}{3} = \frac{11}{6} \times \frac{3}{2} = \frac{11 \times 3}{6 \times 2} = \frac{33}{12} = 2\frac{9}{12} = 2\frac{3}{4}$$

Now Try These

a. $2\frac{1}{2} \div 3\frac{3}{4} = \mathbf{\frac{2}{3}}$

b. $1\frac{5}{8} \div 4\frac{1}{5} = \mathbf{\frac{65}{168}}$

Skill Builders

Multiply and convert to lowest terms:

1. $\frac{1}{15} \times \frac{2}{3} = \mathbf{\frac{2}{45}}$

2. $\frac{3}{20} \times \frac{1}{4} = \mathbf{\frac{3}{80}}$

3. $\frac{4}{11} \times 11 = \mathbf{4}$

4. $\frac{2}{5} \times \frac{5}{6} = \mathbf{\frac{1}{3}}$

5. $\frac{1}{3} \times 24 = \mathbf{8}$

6. $18 \times \frac{1}{6} = \mathbf{3}$

7. $\frac{7}{2} \times \frac{2}{14} = \mathbf{\frac{1}{2}}$

8. $\frac{3}{27} \times \frac{9}{4} = \mathbf{\frac{1}{4}}$

9. $1\frac{3}{10} \times \frac{4}{5} = \mathbf{1\frac{1}{25}}$

10. $4\frac{1}{2} \times \frac{3}{4} = \mathbf{3\frac{3}{8}}$

Divide and convert to lowest terms:

11. $\frac{1}{3} \div \frac{1}{3} = \mathbf{1}$

12. $\frac{1}{12} \div \frac{3}{4} = \mathbf{\frac{1}{9}}$

13. $\frac{7}{16} \div \frac{1}{4} = \mathbf{1\frac{3}{4}}$

14. $\frac{10}{17} \div \frac{2}{3} = \mathbf{\frac{15}{17}}$

15. $\frac{7}{8} \div 2 = \mathbf{\frac{7}{16}}$

16. $32 \div \frac{1}{2} = \mathbf{64}$

17. $3\frac{1}{4} \div \frac{1}{4} = \mathbf{13}$

18. $5\frac{3}{5} \div 2\frac{1}{3} = \mathbf{2\frac{2}{5}}$

19. $1\frac{15}{16} \div 2\frac{4}{5} = \mathbf{\frac{155}{224}}$

20. $1 \div \frac{4}{5} = \mathbf{1\frac{1}{4}}$

Check Yourself

TRY A SHORTCUT: You can use a shortcut to reduce products and quotients to lowest terms. You can reduce as you go along by dividing *opposite* numerators and denominators by their GCF:

$\frac{5}{4} \times \frac{8}{25}$ compare 5 with 25; both can be divided by **5**

compare 4 with 8; both can be divided by **4**

Rewrite the problem:

$$\frac{5}{4} \times \frac{8}{25} = \frac{{}^{1}\cancel{5}}{{}_{1}\cancel{4}} \times \frac{\cancel{8}^{2}}{\cancel{25}_{5}} = \frac{1}{1} \times \frac{2}{5} = \frac{2}{5}$$

This shortcut can help you solve problems quickly without having to multiply large numbers.

Use the shortcut to solve these problems:

1. $\frac{2}{3} \times \frac{3}{2} =$ ***1***

2. $\frac{14}{3} \times \frac{9}{7} =$ ***6***

3. $\frac{6}{8} \div \frac{3}{4} =$ ***1***

4. $1\frac{4}{5} \div \frac{1}{10} =$ ***18***

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Fractions and Decimals: Lesson 1.7

The skills demonstrated in this module are prerequisite skills for lessons in the next module.

Develop Your Understanding

Review all examples. Afterwards, ask for volunteers to come to the board or overhead and work the Now Try These.

Skill Builders

Learners work independently on this section. Have them check their work by using the Answer Key.

Check Yourself

Learners work independently on George's problem. Ask for volunteers to come to the board and show how they solved it.

Closure

Since this is the last lesson of the module, use this time to review all lessons. Allow time for learners to go over their lessons and write down problems to be reviewed. Manage the review according to the needs of each class.

Lesson 1.7
Fractions and Decimals

Learning Objectives

To understand relationships between fractions and decimals
To practice converting fractions to decimals and decimals to fractions
To learn some common fraction/decimal conversions

You have already practiced basic techniques for adding, subtracting, multiplying, and dividing decimals and fractions. Since both decimals and fractions are used to express values that are less than one, it is possible to express many of the same values in fraction or decimal form. Since both forms may be used for many purposes at work, you will need to know how to convert from one form to another in order to compare values.

Develop Your Understanding

Converting Fractions to Decimals

To convert a fraction to a decimal, simply divide the numerator by the denominator. (The bar in a fraction is like a division symbol.)

Example 1 *Convert* $\frac{4}{5}$ *to decimal form.*

$$\begin{array}{r} 0.8 \\ 5\overline{)4.0} \\ \underline{-40} \\ 0 \end{array}$$

Dividing the numerator (4) by the denominator (5) gives a quotient of 0.8.

$\frac{4}{5} = 0.8$

Now Try These

Convert each fraction to decimal form. Round off to the nearest *hundredth* if applicable.

a. $\frac{2}{3}$ = ***0.67*** b. $\frac{4}{7}$ = ***0.57***

Converting Decimals to Fractions

To convert a decimal to a fraction, rewrite the decimal as a numerator without the decimal point; the denominator will be a multiple of 10.

To determine the correct denominator, count the original number of decimal places. The denominator will be a "1" followed by that number of zeros. Note that if the decimal value farthest to the right is in tenths, the denominator will be 10; if that value is in hundredths, the denominator will be 100; if that value is in thousandths, the denominator will be 1000.

Example 2

Convert 0.71 to fraction form.

$\underline{71}$	*Write 0.71 as a numerator, dropping the decimal point.*
$\overline{100}$	*There are 2 decimal places, meaning 71 hundredths, so there will be 2 zeros in the denominator.*
$0.71 = \frac{71}{100}$	*The decimal "71 hundredths" is the fraction "71 over 100."*

After converting a decimal to fraction form, be sure to reduce the fraction to lowest terms.

Example 3

Convert 0.65 to fraction form, and reduce to lowest terms.

$\underline{65}$ *Write 65 as the numerator, dropping the decimal point.*

$\frac{65}{100}$ *Since there are 2 decimal places, the denominator is 100.*

$\frac{65 \div 5}{100 \div 5} = \frac{13}{20}$ *Divide both the numerator and denominator by 5.*

$0.65 = \frac{13}{20}$

Now Try These

Convert each decimal to a fraction. Reduce to lowest terms if applicable.

a. $0.225 = \mathbf{\frac{9}{40}}$

b. $0.3 = \mathbf{\frac{3}{10}}$

Common Conversions

These fractions and decimals are very often substituted for each other. You may want to memorize these:

Fraction	**Decimal Equivalent**
$\frac{1}{10}$	0.1
$\frac{1}{5}$	0.2
$\frac{1}{4}$	0.25
$\frac{1}{2}$	0.5
$\frac{3}{4}$	0.75

Skill Builders

Convert these fractions to decimal form. Round off to the nearest *thousandth* if applicable.

1. $\frac{5}{6}$ = ***0.833***

2. $\frac{3}{5}$ = ***0.6***

3. $\frac{1}{8}$ = ***0.125***

4. $\frac{1}{3}$ = ***0.333***

5. $\frac{9}{10}$ = ***0.9***

6. $\frac{3}{20}$ = ***0.15***

Convert these decimals to fractions, then express in lowest terms.

7. 0.625 = $\mathbf{\frac{5}{8}}$

8. 0.72 = $\mathbf{\frac{18}{25}}$

9. 0.144 = $\mathbf{\frac{18}{125}}$

10. 0.4 = $\mathbf{\frac{2}{5}}$

11. 0.0045 = $\mathbf{\frac{9}{2000}}$

12. 0.02 = $\mathbf{\frac{1}{50}}$

Check Yourself

Read the following. Then solve George's problem.

George's supervisor said, "Someone on the first shift marked these packages differently. I want all the weights marked in fractions."

Package 1 = $\frac{3}{8}$ lb

Package 2 = 0.5 lb

Package 3 = 0.25 lb

Package 4 = $\frac{3}{4}$ lb

How would George rewrite the weights of Package 2 and Package 3?

1. Package 2 = _______ lb $\frac{1}{2}$

2. Package 3 = _______ lb $\frac{1}{4}$

NAME ______________________________

Module 1 Posttest

Directions: *Do your calculations in the space provided.*

1. Which of the following numbers is the largest?

   a. 9.497
   b. 9.4
   c. 9.794
   d. 9

2. Which of the following numbers is the smallest?

   a. 11.99
   b. 12.5
   c. 12
   d. 12.15

3. Round off the decimal number 4.8 to a whole number.

4. Add: 5.546 + 0.3 + 20 =

5. Add: 0.024 + 0.003 + 0.00006 =

6. Subtract: 3 – 2.864 =

7. Subtract: $5.344 - 2.776 =$

8. Multiply: $3.6 \times 35 =$

9. Multiply: $31.27 \times 0.03 =$

10. Divide: $3.6\overline{)17.28} =$

11. Divide: $0.03\overline{)192.6} =$

12. Change the improper fraction to a mixed number.

$\frac{56}{9} =$

13. Change the mixed number to an improper fraction.

$8\frac{2}{3} =$

Reduce all fractions to lowest terms in items 14 through 17.

14. Add: $\frac{2}{5} + \frac{1}{5} =$

15. Add: $\frac{3}{8} + \frac{7}{8} =$

16. Subtract: $\frac{7}{8} - \frac{3}{8} =$

17. Subtract: $\frac{7}{12} - \frac{5}{12} =$

18. What is the least common denominator for $\frac{1}{4}$ and $\frac{2}{3}$?

19. What is the least common denominator for $\frac{1}{3}$ and $\frac{2}{5}$?

Add and simplify items 20 and 21.

20. Add: $\frac{5}{9} + \frac{1}{6} =$

21. Add: $8\frac{5}{8} + 10\frac{1}{4} =$

Subtract and simplify items 22 and 23.

22. Subtract: $\frac{4}{5} - \frac{2}{10} =$

23. Subtract: $1\frac{3}{4} - \frac{2}{3} =$

Multiply and simplify items 24 and 25.

24. Multiply: $\frac{3}{4} \times 7 =$

25. Multiply: $\frac{3}{4} \times \frac{1}{6} =$

Divide and simplify items 26 and 27.

26. Divide: $3 \div \frac{2}{5} =$

27. Divide: $\frac{4}{5} \div \frac{1}{7} =$

NAME ____________________________

Module 2 Pretest

Directions: *Do your calculations in the space provided.*

1. Estimate the total cost of the office supplies listed below. Round to the nearest whole dollar.

   $22.17
   $21.98
   $20.51
   $23.49
   $19.99

2. A coach airline ticket to Centerville is $139 less than a first-class ticket. A first-class ticket to Centerville is $305. How much is a coach ticket?

3. Three-fourths ($\frac{3}{4}$) of the employees at XYZ Graphics gave to the People's Fund this year. If 1,200 employees work at XYZ Graphics, how many gave to the People's Fund?

4. What is the value of X?

   $$\frac{X}{15} = \frac{3}{9}$$

5. A recipe for mixing paste calls for $\frac{3}{4}$ cup of water to make half a jar of paste. How much water would you use to make 2 full jars of paste?

6. Find the average of these numbers:

20, 14, 13, 16, 17

7. Use the chart to calculate the *average daily number* of utility customer complaints for March 14–18:

Date	**Number of Complaints**
Monday, March 14	82
Tuesday, March 15	63
Wednesday, March 16	118
Thursday, March 17	91
Friday, March 18	66

8. What is the perimeter of a square panel if the length of one side is 5 inches?

9. In order to make Restful Green paint, you must mix the following colors:

3 parts yellow
2 parts blue
1 part white

How many jars of blue paint would you need if you used 4 jars of white?

10. If you can get exactly 12 gallons of gas for $18, how much gas could you get for $15?

11. What is 8% of 280?

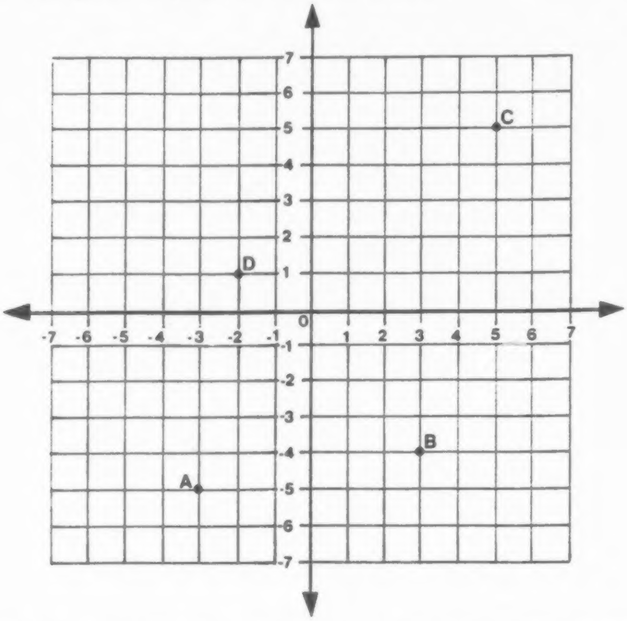
12. Express this fraction as a percentage:

$$\frac{3}{20} = ?\%$$

13. Of a total of 1,625 wiring boards manufactured on a certain day, 4% had defects. How many boards had defects?

14. A special project for ABC Computers has a total budget of $8,000, with $1,200 of the budget to be spent on supplies. What percentage of the budget will be used for supplies?

Use the coordinate system grid shown below to answer questions 15 and 16. Each unit on the grid represents one inch.



15. What are the coordinates of each of the following points?

A ________

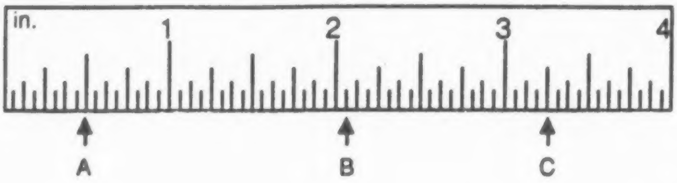
B ________

C ________

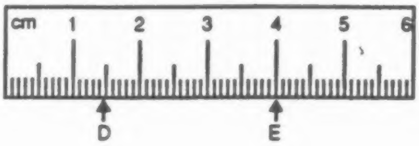
D ________

16. If point C were moved 2 inches to the left, what would be its new coordinates? (Remember, each unit on the grid is one inch).

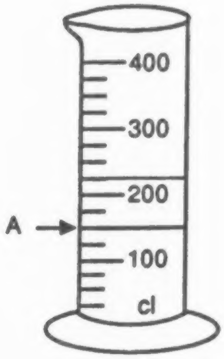
17. Using the *standard* ruler shown below, what measurements are marked by the letters A, B, and C?



18. Using the *metric* ruler below, what is the distance between point D and point E?


cm
1
2
3
4
5
6
D
E

19. How much liquid is in the cylinder shown below (filled to the line marked "A")?


400
300
200
100
cl
A

20. 1 centimeter (cm) = 10 millimeters (mm) How many millimeters are there in 2.8 centimeters?

21. $16 - 18 =$

22. $21 \times (-2) =$

23. $6 - (-3) =$

24. What is the value of x?

$x - 25 = 60$

25. What is the value of y?

$3y = -6$

26. In a certain department, three times as many people work full-time as work part-time. If the total number of employees in the department is 28, how many work part-time?

27. When a certain number is divided by 8, the quotient is also 8. What is that number?

28. What is the value of y?

$$\frac{2y}{6} = 4$$

29. Use this formula for finding distance to calculate the miles driven on a weekend trip.

Distance = Rate x Time

where Rate = 45 miles per hour
Time = 8 hours

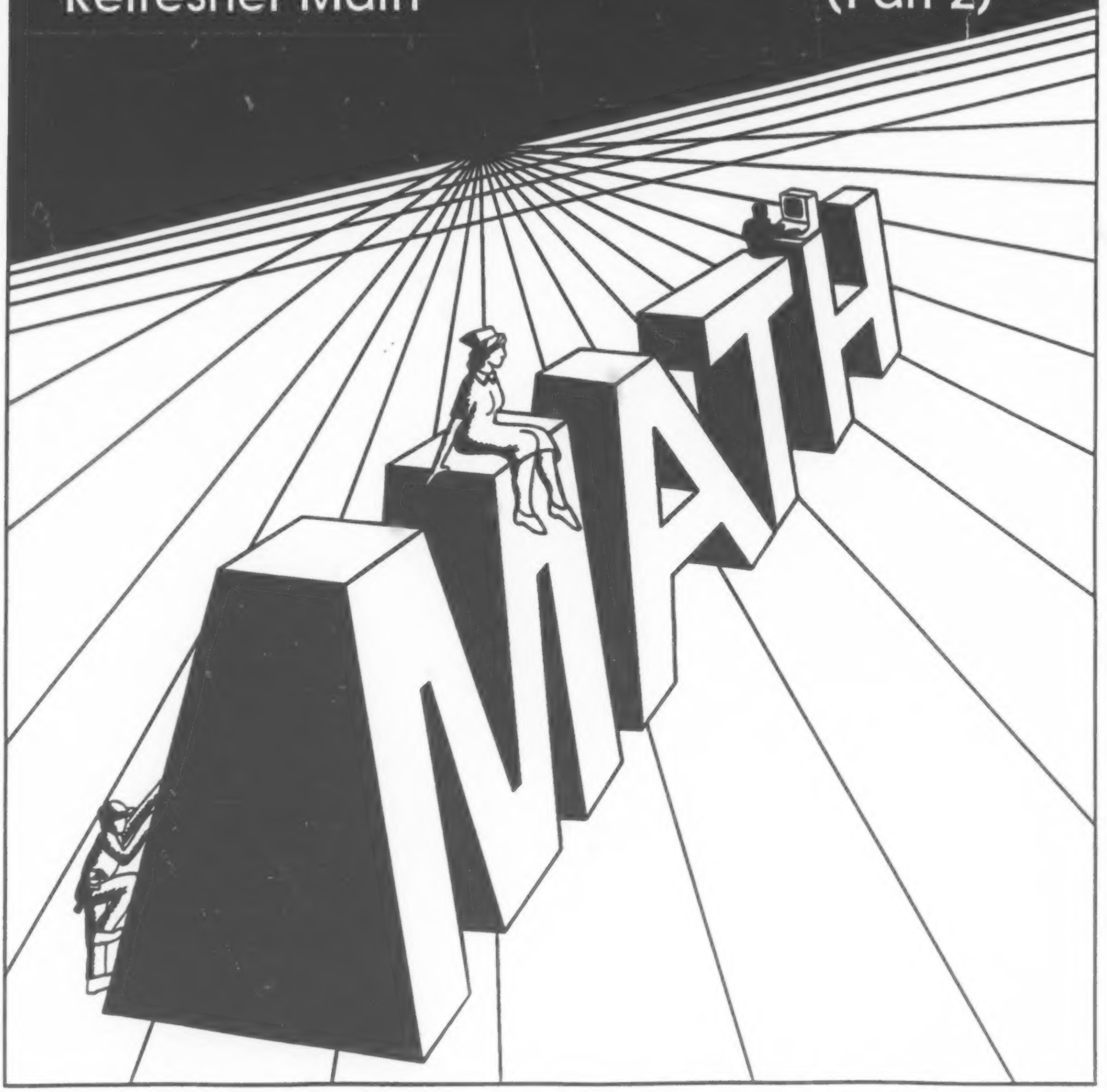
30. Use this formula to calculate percentage yield for computer parts. Your answer must be a percent.

$$\text{Percentage Yield} = \frac{\text{quantity out}}{\text{quantity in}}$$

where quantity out = 57
quantity in = 60

Module 2

Refresher Math (Part 2)



MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Solving Word Problems: Lesson 2.1

This lesson provides a strategy for solving word problems involving the four basic operations.

Develop Your Understanding

Read and discuss all aspects of the four-step strategy suggested. Review all examples in the lesson. Research findings on problem solving indicate that **experience** in solving problems is an important ingredient in learning the strategy. Thus, you may want to give your learners additional problems.

Write on Board
A mind is like a computer. Sometimes you need to shut it down and boot it up again.

After the break, continue working as a group independently.

Skill Builders

Learners work independently on this section. Have them check their work by using the Answer Key. The following could be put on an overhead or on the board while learners are working on Skill Builders.

1. **Skim** and **scan** the problem to get a general overview of "the problem."
2. Write or jot down every piece of information that is important. Draw pictures, or **label** each piece of information.
3. Try different tactics:
   a. **Check** to see if there is a relationship among facts.
   b. Think of formulas that could be used.
   c. Try working the problem from the end to the beginning.

d. **Ask** these questions:
What do I need to know to get the answer?
Do I need to reduce, etc.?
Is my answer reasonable?

Check Yourself

Follow the same procedure as in Skill Builders.

Lesson 2.1
Solving Word Problems

Learning Objectives

To develop a strategy for solving word problems and/or problems that arise in the workplace

Just the mention of word problems is enough to make some people uncomfortable. In this lesson, we are going to give new meaning to this task by teaching a strategy that will give you confidence in solving these puzzles, mysteries, and challenges.

Develop Your Understanding

Word problems are similar to instructions in that they must be read very carefully. Most people usually focus on the numbers instead of the "story." You must learn to *visualize* what is described in the problem, and then apply this four-step strategy.

1. Define the question.
2. Determine key information.
3. Determine mathematical operation or operations to be used.
4. After solving, check to see if your answer is reasonable.

Defining the Question

Before you can define the question, you have to find it! In other words, make sure of what the problem is asking you to do before you try to solve it. What type of unit should be attached to the answer? Should it be in hours, ounces, number of people, or some other measure?

Most questions can be found at the end of each word problem, but sometimes the question does not have a question mark. For example, the question may be stated, "Find how much..."

Determining Key Information

After finding the question, the next step involves separating the *key information* from unnecessary information. Some problems state more information than you need to solve the problem. Focus only on the information that is needed. The key information consists of the numbers and the words or symbols that are with the numbers. For example, if the shipping department ordered 12 boxes of tape at $3.00 a box and you were asked to find the total cost, you might analyze the problem as follows:

QUESTION: What is the cost of 12 boxes of tape?

KEY INFORMATION: 12 boxes and $3.00 a box

ANSWER IN (UNITS): dollars and cents

Determining Mathematical Operation or Operations to Be Used

The third step in solving a word problem is determining the mathematical operation or operations to be used. In order to do this effectively, you can find the key words, make diagrams, or do both. Listed below are some important key words for the four fundamental mathematical operations.

ADDITION

total
plus
add
sum
more
increase
altogether
both
combined
in all
additional

SUBTRACTION

difference
fewer
decreased
minus
reduce
left
remainder
any type of *-er* comparison

MULTIPLICATION

of
times
at
by
as much
product
per
twice

DIVISION

quotient
divided evenly
split
shared
out of
every

These key words can help you decide which operation to use. Sometimes, however, a key word may indicate the *opposite* operation. For example, in order to find the *increase* in something, you may have to *subtract*. Remember:

> Addition and subtraction are opposites.
> Multiplication and division are opposites.

Multiple-Step Operations

In certain situations, you will need to use more than one operation. For instance, you may be asked to combine amounts and find the difference. This requires you to add and then subtract. Once again, you must visualize the problem and ask yourself, "How can I get there from here?"

Checking the Answer

Once you arrive at an answer, check to see if it looks reasonable. In order to get an exact check, reverse the operations that led you to the answer and try to get the original number back. For example, if you used division and then addition to solve a problem, you would use subtraction and then multiplication to check it.

Example 1

Judy is reading a 324-page training manual. She has already read 112 pages. If she reads 53 pages a day, how many days will it take to finish reading the manual?

QUESTION:	*How many days until Judy finishes?*
KEY INFORMATION:	*324 pages total; 112 pages already read; reading rate is 53 pages per day*
ANSWER IN (UNITS):	*days*
MATHEMATICAL OPERATIONS:	*subtraction, then division*
CHECK:	*multiplication, then addition*

After filling out the chart, you might say to yourself: The number of pages in the book minus the number already read is the number of pages left to read. I need to know how many days it will take to read the remaining pages if Judy reads 53 pages per day. So, I need to **set up the operation** as follows:

Subtract:	*Then divide:*
324 − 112 = 212	212 ÷ 53 = 4 days

Check:

Multiply:
53 pages × 4 (days) = 212

Then add:
212 + 112 = 324

Now Try These

a. Irene assembled parts for a special display at the HAK Computer Show. One part comes in four sections. Section A is 4.862 centimeters, Section B is 7.899 centimeters, Section C is 3.2 centimeters, and Section D is 0.3 centimeter. What is the total length when all four sections are combined?

QUESTION:

KEY INFORMATION:

ANSWER IN (UNITS):

OPERATION:

CHECK:

16.261 ANSWER:

b. Three-fifths ($\frac{3}{5}$) of the employees at XI signed up to contribute to United Help. How many employees out of 1,200 contributed?

QUESTION:

KEY INFORMATION:

ANSWER IN (UNITS):

OPERATION:

CHECK:

720 ANSWER:

c. The ABC Computer Co. has 892,000 employees working in 50 branches throughout the United States. If 21,000 employees are laid off, how many will remain?

QUESTION:

KEY INFORMATION:

ANSWER IN (UNITS):

OPERATION:

CHECK:

871,000 ANSWER:

---

d. 300 computer manuals were packed in 50 boxes. Joyce was told to divide the manuals evenly. How many manuals were put in each box?

QUESTION:

KEY INFORMATION:

ANSWER IN (UNITS):

OPERATION:

CHECK:

6 ANSWER:

Skill Builders

For each problem, look for the ***key words.*** *Then, solve the problem and check it. Be sure to read* ***carefully.***

1. Lance works 6 hours for XI. On weekends he earns an extra $200.00 working on race cars. What other information do you need to find his total income?

   Lance's hourly wage

2. The delivery truck driver recorded the following trips:

Dripping Springs	7.1 miles
Blanco	16.4 miles
Johnson City	15.8 miles
Wimberley	12.9 miles

   What was the total mileage for the four trips?

   52.2 miles

3. T.J. uses 30 rolls of tape per week, and he must fill a requisition order for 6 weeks. Rolls of tape come in packages of 3. How many packages does he need to order?

   60 packages

4. Peggy planned to spend $6\frac{1}{2}$ hours organizing the kitting stockroom. She has worked $1\frac{3}{4}$ hours. How many hours does she have left to work?

   $4\frac{3}{4}$ hours

5. In a memo, Joe was told to stack boards that were $\frac{1}{2}$" thick and 11' long. The stack should be 30" high. How many boards share the 30" stack?

   60

6. UR2 Graphics gave a $250,000.00 bonus to their employees. The amount was shared evenly among 750 employees. Find the amount each employee received.

   $333.33

7. In 1990, the ABC Computers had sales of $2,143,021. For 1991, their sales totaled $4,000,021. What is the increase in sales?

   $1,857,000

8. The personnel office reported 60 employees absent from work on Friday and 90 on Monday. There are 1,200 employees. Which day had less than 0.06 of the total number of employees absent? Show all work.

Friday

9. The trimmer at UR2 Graphics was told to trim the Moreland Manual $\frac{1}{2}$". He was later told to take another $\frac{1}{4}$" off. The inspector was then told that another $\frac{1}{8}$" should be taken off. How much had been trimmed altogether *before* the inspector was told of the change?

$\frac{3}{4}$"

10. Shakir was told to cut a 62" tape into three 18 $\frac{1}{3}$" strips. How much tape will be left?

7'

11. Three-eighths ($\frac{3}{8}$) of the people on the Orange Line worked on Saturday. One-fourth ($\frac{1}{4}$) of the people on the Orange Line worked on Sunday. *What further information would you need* to find the total number of employees not working on these days?

number of employees; also, whether some of the same people worked both Saturday and Sunday

12. UR2 Graphics gives an employment test that takes 2 hours. The math section takes $\frac{1}{5}$ of the time. How long is the math test?

24 minutes

13. John's supervisor told him to cut $1\frac{1}{16}$" off the flyer he was preparing for "Taco to Munch." The flyer was originally $10\frac{1}{4}$" long. How many inches were cut off? *(Caution: Read carefully.)*

$1\frac{1}{16}$"

14. In a union poll, $\frac{3}{5}$ of the 500 members agreed with Charles, and $\frac{2}{5}$ agreed with John. Find the number of people who agreed with John.

200

15. Laverne is ordering paper for the copying machine. Paper comes 500 sheets to a ream and 12 reams to a case. If a case costs $90.00, how much does the paper cost per sheet? (Be sure to round your answer to the nearest cent.)

2¢

16. Total prize money for the XI Big Bash Tennis Tournament was $93,950.00. Smash Wilson won first place and collected $33,725.00. The remaining money was divided equally among 5 other players who had the best records of the 214 entrants. How much did each of these other players receive?

$12,045.00

17. Edwin was told to find the cost of replacing 18 windows at XI. If 3 windows can be installed in one hour, and each window costs $45.37, how much will it cost the company to replace the windows if the cost of labor is $9.00 per hour?

$54.00	***labor***
816.66	***windows***
$870.66	***total***

18. XI has a wall made up of 564 panels. It takes a window washer 5 minutes to clean each panel. How many days will it take 2 washers working 8-hour days to wash all the panels? Round off to the nearest whole number of days.

3 days

Check Yourself

1. Make a diagram, picture, or mnemonic device to help you remember the four-step strategy for solving word problems.

Answers will vary.

2. Before each key word, write the symbol of the mathematical operation usually used when solving word problems. The first one is done for you.

+	total
×	of
×	by
−	difference
−	left
+	every
÷	split
+	combined
×	product
÷	shared
+	altogether
+	sum

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Estimating and Averaging: Lesson 2.2

The major skills addressed in this lesson are those of estimating and averaging. In estimating answers, learners must be fluent with the skill of rounding. Therefore, they need to understand the significance of different place values given different situations.

Develop Your Understanding

Before learning estimation, learners must master the skill of rounding.

Write on Board

1. *Determine place value.*
2. *Look at the digit to the right of that place value.*
3. *Follow the rule for rounding:*
   If the digit is a 5 or above, you add one to the next place value to the left and drop all digits to the right.
   If the digit is a 4 or below, you leave the next place value to the left as it is and drop all digits to the right.

Review rules for averaging before independent work.

Skill Builders

Learners work independently on this section. Have them check their work by using the Answer Key in the back of the book.

Closure

If time allows, put candy in a large jar. Ask learners to estimate the number of pieces of candy in the jar. Allow time for discussion of possible ways to determine the amount of candy.

Lesson 2.2
Estimating and Averaging

Learning Objectives

To practice estimating answers using various operations
To practice the skill of averaging

There are many times in the workplace when you add or subtract numbers or make some other calculation. It is always wise to estimate the answer, even when using a calculator. If a number were accidentally entered incorrectly, an estimate of the answer would quickly point out the problem and allow you to correct your figures before you acted on an incorrect answer. Averaging is another useful skill and is often used to track defect or scrap rates, production rates, or defects.

Vocabulary

averaging—to measure the central tendency of a group of numbers; i.e., to find out what the midpoint is for a group of numbers.
estimating—to figure the general answer to mathematical operations by rounding numbers off to the nearest whole, tenths, or hundredths place, etc.

Develop Your Understanding

Any time you are making a mathematical calculation, whether you are using a calculator or not, it is wise to estimate the answer. Numbers are often entered incorrectly into a calculator, or batteries may be running down, and an answer on the calculator can be wrong. Estimating is also useful in everyday life when buying something or expecting change back.

Estimating

When estimating, you must first determine the place value. For example, if you are dealing with money, you may decide to round off to the nearest whole dollar.

Remember, to round a number, you look at the digit to the right of the place you wish to round to. Then:

1. For values of 4 or below, round *down* to 0. Drop all digits to the right.
2. For values of 5 or above, add 1 to the next place value to the left. Drop all digits to the right.

Example 1

Suppose the supply totals for each week of the last month were as follows:

$9.24
$10.75
$4.50
$5.49

Round each number to the nearest whole dollar in this manner:

$9.24
↑

Look at the digit to the right of the decimal. It is a 2, so according to the rule, leave the number as it is and simply drop the 2 and the 4. The number would then become $9.00.

For $10.75, look at the digit next to the decimal. It is a 7. Following the rule, add 1 to the whole number and drop the end digits. The number is now $11.00.

$4.50 becomes $5.00, and $5.49 becomes $5.00.

Take the estimates and add them:

$$\begin{array}{rlr} & \$ & 9.00 \\ & \$ & 11.00 \\ & \$ & 5.00 \\ + & \$ & \underline{5.00} \\ & \$ & 30.00 \end{array}$$

The total estimate is $30.00.

The actual answer is:

$$\begin{array}{r} \$9.24 \\ 10.75 \\ 4.50 \\ \underline{+\ 5.49} \\ \$29.98 \end{array}$$

The estimate was only off by 2 cents.

If you needed to estimate the combined width of panel layers produced on Monday's first run, you might have the following figures:

$$\begin{array}{r} 1.237 \\ 0.987 \\ 2.119 \\ 1.005 \end{array}$$

First, decide what place to round them to. In this case, it would be best to choose the tenths place. Next:

1. Look at the digit to the right of the tenths place.
2. If it is below 5, leave the number as it is and simply drop the end digits.
3. If it is a 5 or above, add 1 to the number and drop the end digits.

↓

For example, with the number 1.237, we look at the digit to the right of the 2 (which is in the tenths place). This number is a 3, so leave the number as it is and simply drop the digits to the right: drop the 37, and 1.237 becomes 1.2.

With 0.987, look at the number to the right of 0.9; it is an 8. Therefore, add 1 to the number 9 and drop the digits to the right:

$$\begin{array}{ll} 0.9 & \text{drop the 87} \\ \underline{+\ 1} & \\ 1.0 & \text{So 0.987 becomes 1.00.} \end{array}$$

↓ ↓

2.119 becomes 2.1. And 1.005 becomes 1.0.

Add your estimates as follows:

$$\begin{array}{rr} & 1.2 \\ & 1.0 \\ & 2.1 \\ & +\underline{1.0} \\ \textit{The total estimate is:} & 5.3 \end{array}$$

The actual width for the combined layers is:

$$\begin{array}{r} 1.237 \\ 0.987 \\ 2.119 \\ \underline{+1.005} \\ 5.348 \end{array}$$

Now Try These

Estimate the following costs of plating chemicals for one day. Round to the nearest whole dollar.

a. $10.24	b. $9.23	c. $10.68
15.79	11.89	9.34
12.36	12.45	11.02
15.05	8.57	9.99
$53.00	***$42.00***	***$41.00***

The actual costs were:
a. $53.44
b. $42.14
c. $41.03

How close were your estimates?

Averaging

Averaging is another important skill in the workplace. It is useful when looking at overall patterns of cost, production, temperature, or other measures. It gives you the midpoint of the numbers you are averaging. To average:

1. Add to get the total of the figures you are considering.
2. Then divide that total by the number of figures that you added.

Example 2

Suppose you want to know the average temperature of the plating bath on Monday morning. Add the temperatures taken each hour and then divide by the number of temperatures.

TIME	TEMPERATURE
7:00 a.m.	99°
8:00 a.m.	100°
9:00 a.m.	103°
10:00 a.m.	98°
11:00 a.m.	101°

Add 99° + 100° + 103° + 98° + 101°, to get 501°.

Since you added 5 *temperatures* to get that total, *divide the total by 5:*

501° ÷ 5 = 100.2°

100.2° was the average temperature for the plating bath on Monday morning.

Now Try These

Find the average daily cost of supplies for the lamination area for one week.

Monday:	$100.25
Tuesday:	$57.50
Wednesday:	$75.00
Thursday:	$105.75
Friday:	$200.50

$107.80

If you estimated the average, what would you estimate it to be?

$108.00

Skill Builders

1. Estimate the total cost for these items: Round to the nearest whole dollar.

   $2.25
   10.99
   3.49
   5.59

   $22.00

2. Estimate the total production numbers by rounding to the nearest hundred. The following are the totals for each day: 127, 228, 109, 179, 188.

   800

   Then calculate the actual total production, and compare with your estimate.

   831

3. Your supervisor is concerned that the cost of supplies in your area has increased greatly in the last month. Previous monthly costs were about $200.00. Given the following supply costs for each week, what would you estimate is your current monthly supply spending? Round to the nearest whole dollar.

week 1	$175.23
week 2	25.15
week 3	50.99
week 4	26.75

$278.00

4. Your supervisor takes everyone in your area out to lunch. The cost for each person is as follows. Estimate the total bill *not including* tax and tip. (Round to the nearest dollar.)

$4.99 $3.50 $2.99 $3.25 $5.75

$21.00

5. What is the average of the following numbers? Round *your answer* to the nearest hundredth.

23, 19, 18, 15, 20, 26

20.17

6. What is the average of these cost figures?

$11.12, $12.28, $10.60, $12.25, $13.75

$12.00

7. What is the average number of pencils being used weekly if the numbers used for each of the last four weeks were 19, 9, 10, and 12?

12.5

8. The air temperature for the yellow room must be kept within a certain range. Lately, it may be going too far out of acceptable limits. Find the average temperature for the room for the past five days. Daily temperatures were: 69°, 72°, 67°, 73°, 74°.

71°

9. What is the average daily production rate for your area for this week if the number of layers produced each day is as follows?
   150, 25, 200, 78, 129

   116.4

10. Mary went to the company store to purchase several items. They cost \$29.99, \$9.99, \$4.49, \$3.29, and \$12.79. She handed \$70.00 to the clerk. Estimate the total cost of the items, and then estimate how much change she should receive in return.

    Estimated cost = \$60.00

    Estimated change = \$10.00

11. You enter the following numbers into a calculator and get the answer shown:

$$\begin{array}{r}104\\302\\459\\\underline{398}\\4012\end{array}$$

    What would you estimate the answer to be (to the nearest hundred)?

    1300

    Were the numbers entered in the calculator correctly? ***No***

12. There is a daily average of 13.5 people in Area B each week, and they need two sets of safety gloves each day for each person. Estimate how many sets of safety gloves they need each week.

    140

13. ABC Computers uses about 15.2 lb of dry chemicals and 10.7 lb of liquid chemicals each week on the plating line. Estimate the total amount of chemicals used on the line over a 4-week period.

104 lb

14. If George was absent for the following number of days for each of the last 6 months, what is his average absentee rate for those 6 months?

0, 4, 2, 8, 1, 3

3 days

15. The following temperature readings were taken from the machine in the lamination area during one week. Estimate the average temperature of the machine for this period to the nearest whole degree.

200.51°, 175.25°, 224.90°, 208.62°, 100.45°

182°

MEMO

TO: The Instructor
FROM: The Students
SUBJECT: Working with Ratios: Lesson 2.3

This lesson introduces learners to the concepts of ratio and proportion, and their practical uses, with a focus on ratios. If learners become confused about the difference between a ratio and a fraction, you may explain that any fraction is a type of ratio, but viewing it as a ratio emphasizes a comparison between the two numbers involved.

Develop Your Understanding

Writing Ratios

Learners should practice all three methods of writing ratios even though the first and third methods are most common.

Write on Board
The bar in a fraction represents a division symbol.

Putting Ratios in Lowest Terms

Ratios that are represented by improper fractions are not rewritten as mixed numbers because ratios help to make a clear comparison between the two numbers, rather than to express the amount as a single number.

Calculating Equivalent Ratios

The process of calculating equivalent ratios is the same as the process of calculating equivalent fractions.

Solving Problems with Ratios

Ratios can also be written with a colon or division symbol. This may be helpful for those who are having trouble distinguishing them from fractions.

Skill Builders

Learners work independently on this section. Have them check their work by using the Answer Key in the back of the book.

Check Yourself

Learners must first calculate the number of days worked by each employee by multiplying the number of months worked by 23, then set up a ratio to compare to the 1:200 ideal ratio. Learners may try this on their own, or you may use it to introduce the next lesson.

Lesson 2.3
Working with Ratios

Learning Objectives

To write ratios based on verbal or written instructions
To practice computing equivalent ratios
To solve word problems using ratios

Ratios are methods of making numerical comparisons. They are useful in a variety of ways, from expanding the amount of liquid yielded by mixing chemicals to making accurate price comparisons. It is important to have an understanding of ratios before you move on to working with percents.

Vocabulary

ratio—the ratio of one number to another is the first number divided by the second. A ratio is also a way of comparing two numbers or showing relationships between them.
equivalent—equal in value

Develop Your Understanding

Writing Ratios

A *ratio* is one number or amount divided by a second number or amount. Ratios can be written in three ways. For example, to show the ratio of 6 to 12, you can:

1. use a colon	$6:12$
2. use a division symbol	$6 \div 12$
3. use a fraction	$\frac{6}{12}$

Often, when working with ratios, writing them as fractions is preferred. A ratio that is written as a fraction may be treated as any other fraction. Using a colon is also a very common way of writing ratios. Using a division symbol is seen less frequently.

Example 1: What is the ratio of 8 to 10?

Write the ratio as a fraction, with the first number given as the numerator (on the top), and the second number given as the denominator (on the bottom).

$\frac{8}{10}$ is the ratio of 8 to 10.

When a ratio shows a relationship using a unit of measurement such as miles, pounds, or gallons, it is best if both the numerator and the denominator are expressed in the same type of unit. This makes it easier to compare amounts.

Example 2: Show the ratio of 3 inches to 1 foot.

Here, you can convert feet to inches so that the unit of both numbers will be the same.

1 foot = 12 inches

The ratio of 3 inches to 1 foot is the same as the ratio of 3 inches to 12 inches, which is:

$$\frac{3}{12}$$

Now Try These

Write each of these ratios as a *fraction.* Both numbers in the ratio must be in the same type of unit.

a. 3 pounds to 17 pounds $\frac{3}{17}$

b. 2 inches to 5 inches $\frac{2}{5}$

c. 180 miles to 55 miles $\frac{180}{55}$

d. 45 minutes to 1 hour $\frac{45}{60}$

e. 2 pounds to 9 ounces (*Note:* 1 pound = 16 ounces) $\frac{32}{9}$

Putting Ratios in Lowest Terms

Fractions are most properly expressed in lowest terms. The same is true of ratios.

Example 3: Reduce the ratio $\frac{12}{16}$. *to lowest terms.*

First, you must find the greatest common factor (GCF) of 12 and 16. The GCF is the largest number that can be divided evenly into both parts of the fraction.

The greatest number that can be divided into both 12 and 16 is 4.

Next, divide both the numerator and the denominator of the fraction by 4 to obtain an *equivalent fraction:*

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

The fraction $\frac{3}{4}$ is the same as the fraction $\frac{12}{16}$ expressed in lowest terms. Therefore, we can say that $\frac{12}{16}$ and $\frac{3}{4}$ are *equivalent ratios.*

Ratios that are written as improper fractions, where the numerator is larger than the denominator, are NOT usually rewritten as mixed numbers.

Now Try These

Put each of these ratios in lowest terms:

a. 28:14 = ***2:1***

b. $\frac{6}{30} = \mathbf{\frac{1}{5}}$

c. 10 ÷ 25 = ***2 ÷ 5***

Calculating Equivalent Ratios

In the section above, you learned how to find one kind of equivalent ratio by putting a ratio into lowest terms. Each ratio has many possible equivalent ratios. All that is needed to find an equivalent ratio is to use multiplication or division to perform the same operation on both the numerator and denominator.

Caution: DO NOT use addition or subtraction to calculate equivalent ratios!

Example 4: Calculate five equivalent ratios for the ratio $\frac{4}{6}$.

1. First, put the $\frac{4}{6}$ into lowest terms by dividing each part of the ratio by 2:

   a. $\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

2. Next, use multiplication to calculate 4 more equivalent ratios:

   b. $\frac{4 \times 2}{6 \times 2} = \frac{8}{12}$

   c. $\frac{4 \times 3}{6 \times 3} = \frac{12}{18}$

   d. $\frac{4 \times 4}{6 \times 4} = \frac{16}{24}$

   e. $\frac{4 \times 5}{6 \times 5} = \frac{20}{30}$

Finally, we have 6 ratios: $\frac{4}{6}$ and 5 equivalent ones:

$$\frac{2}{3} = \frac{\mathbf{4}}{\mathbf{6}} = \frac{8}{12} = \frac{12}{18} = \frac{16}{24} = \frac{20}{30}$$

Now Try These

Calculate any *two* equivalent ratios for each ratio given. ***Answers will vary.***

a. $\frac{5}{6} = \mathbf{\frac{10}{12} \quad \frac{15}{18}}$

b. $\frac{10}{8} = \mathbf{\frac{20}{16} \quad \frac{5}{4}}$

c. $\frac{3}{6} = \mathbf{\frac{1}{2} \quad \frac{2}{4}}$

Solving Problems with Ratios

You can solve word problems using ratios. Following these steps:

1. Write the ratio *in words* first. *Do not omit this step!*
2. Find numbers to substitute for the words in the ratio. Sometimes, you will have to do calculations to find the numbers.
3. Calculate an equivalent ratio that solves the problem.
4. Put the answer in lowest terms.

Example 5

Employees working on an assembly line produced 800 panels last week. Of the total number of panels produced, 24 were defective. What is the ratio of defective to nondefective panels produced?

Begin by asking yourself: *What ratio do I need?* Then write the ratio in words:

$$\frac{\textit{defective panels}}{\textit{nondefective panels}}$$

In order to write the ratio in numbers, you will need to know how many of the 800 panels produced were NOT defective. You can find this number by subtracting the number of defective panels from the total number produced:

800 total panels – 24 defective panels = 776 nondefective panels

Now you have both of the numbers you need. The ratio of defective to nondefective panels is 24:776 or $\frac{24}{776}$:

$$\frac{\textit{defective panels}}{\textit{nondefective panels}} = \frac{24}{776}$$

Since both 24 and 776 may be divided by 8, this ratio may be most simply expressed as follows:

$$\frac{24 \div 8}{776 \div 8} = \frac{3}{97}$$

Example 6

In order to make 2 servings of oatmeal, you must use 1 cup of oats and 2 cups of water. How much of each ingredient would you need to make 8 servings of oatmeal?

First, write the ratio of oats to water in the recipe for 2 servings:

$$\frac{\textit{oats}}{\textit{water}} = \frac{1}{2}$$

Next, calculate an equivalent ratio that will supply enough oatmeal for 8 servings. Since the first ratio gives enough oatmeal for 2 servings, divide 8 by 2 to get the number of 2-serving recipes you would need to make 8 servings:

8 ÷ 2 = 4. Therefore, you will need *4 recipes of oatmeal to get 8 servings.*

Multiply both parts of the ratio $\frac{1}{2}$ by 4 in order to obtain an equivalent ratio.

$$\frac{\textit{oats}}{\textit{water}} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

You will need *4 cups of oats* and *8 cups of water* to make 8 servings of oatmeal.

Example 7

When more than two ingredients or components are involved, ratios may still be used.

In order to mix a batch of chemicals, use 1 part Chemical A to 3 parts Chemical B to 2 parts Chemical C. If you are using 6 gallons of Chemical B, how many gallons of Chemical C should you use?

The relationship of the three chemicals can be shown as a three-way ratio:

The ratio of Chemical A to Chemical B to Chemical C is 1:3:2.

Here, we are only concerned with Chemicals B and C. The ratio of B to C is 3:2. Write this ratio as a fraction:

$$\frac{\text{Chemical B} =}{\text{Chemical C} =} \quad \frac{3}{2}$$

Next, you must find an equivalent ratio where the amount of Chemical B is equal to 6.

$$\frac{3}{2} = \frac{6}{?}$$

Find the number that 3 must be multiplied by to get 6. You can do this by dividing 6 by 3:

$$6 \div 3 = 2$$

Next, multiply the denominator, 2, by this number to get the number of gallons that will give you an equivalent ratio:

$$\frac{3 \times 2}{2 \times 2} = \frac{6}{4}$$

$$\frac{\text{Chemical B} =}{\text{Chemical C} =} \quad \frac{6}{4}$$

Therefore, you will need *4 gallons of Chemical C.*

Skill Builders

Skill Practice

Write each ratio in three ways, using a colon, division symbol, and fraction.

1. You have 5 apples and 3 oranges. Write the ratio of *oranges to apples.*

   3:5 ***3 ÷ 5*** $\frac{3}{5}$

2. There is one secretary at the plant for every 18 production workers. Write the ratio of *secretaries to production workers.*

   1:18 ***1 ÷ 18*** $\frac{1}{18}$

3. Yesterday, Fred worked on 25 panels that were good and 3 that were defective. Give the ratio of *defective panels to total panels worked.*

   3:25 ***3 ÷ 25*** $\frac{3}{25}$

Write the following ratios as fractions. Use the same units for both parts of the fractions.

4. 46 to 1

   $\frac{46}{1}$

5. 8 days to 1 week

   $\frac{8}{7}$

6. 2 pounds to 32 ounces

   $\frac{32}{32}$

7. 2.5 inches to 5 inches

   $\frac{2.5}{5}$

8. Reduce each of these ratios to lowest terms.

   a. $\frac{6}{9}$ $\frac{2}{3}$ b. $\frac{24}{8}$ $\frac{3}{1}$ c. $\frac{15}{36}$ $\frac{5}{12}$

9. Replace each question mark to complete an equivalent ratio for each ratio shown:

a. $\frac{2}{6} = \frac{\mathbf{1}}{3}$

b. $\frac{7}{21} = \frac{1}{\mathbf{3}}$

c. $\frac{30}{\mathbf{8}} = \frac{15}{4}$

10. Compute 5 equivalent ratios for the ratio 3:1.

Answers will vary.

Examples are

6:2 ***9:3*** ***12:4*** ***15:5*** ***18:6***

Word Problems

Write each ratio in words; then, solve the problems. Reduce all ratios to lowest terms in your answers.

11. A certain plant has 1,230 employees, *including* 210 managers. What is the ratio of *managers to other employees* at the plant?

7:34

Use the chart below to answer questions 12 -14.

The following chart shows how much of each ingredient is required in order to make a batch of 12 pancakes:

Pancake Mix	Milk	Eggs	Yield
1 cup	$\frac{1}{2}$ cup	1	12 pancakes

12. How many eggs would you use if 2 cups of milk were used?

    4 eggs

13. What is the ratio of *pancake mix to milk?* Be sure that both parts of the ratio are whole numbers.

    $1:\frac{1}{2}$ *or 2:1*

14. What is the ratio of *pancakes to eggs* in the recipe?

    12:1

15. A 50-pound bag of powder costs \$15, while a 75-pound bag of the same powder costs \$25. What is the ratio of *cost to weight* in each case? Are the two ratios equivalent?

    50-lb bag = \$15:50 lb
    75-lb bag = \$25:75 lb

    No, they are not equivalent.

16. Out of a total of 2,250 employees at Gulpo Drink Company's Florida plant, 25 are currently on leaves of absence (LOA). What is the ratio of *total employees to employees on LOA?*

    90:1

17. ABC Computers has a goal of achieving a ratio of *6 defects for every 1,000 computer chips produced.* Out of a sample of 500 chips inspected, 4 were defective. Did ABC achieve its goal?

    No; the actual ratio was equivalent to 8 out of 1,000.

18. A new printing company had expenses of $68,000 in its first month and income of $204,000. Find the ratio of *expenses to income* for the company's first month.

    1:3

19. A mixture of paint has a ratio of *two parts of yellow paint to 4 parts of blue.* If a particular batch of this paint has 8 gallons of yellow paint, how much blue paint is in the batch?

    16 gallons

20. One pound of laundry soap sells for $2.40. Find the *cost per ounce* and write this as a ratio with cost as the numerator and ounces as the denominator. (*Hint:* 1 pound = 16 ounces.)

    15¢ per ounce or $0.15:1 ounce

Check Yourself

Plink Industries has a rule that each employee should have at least *1 day of training for every 6 months worked.* Which of these employees has not had enough training, according to this rule? Use equivalent ratios to solve this problem.

Clarabelle has worked for the company for 12 years and has had 25 days of training.

Roderick has worked for the company for 6 years and 3 months and has had 8 days of training.

Grace has worked for the company for 2 years and 2 months and has had 5 days of training.

> ***Roderick has not had enough training. (Clarabelle should have had at least 24 days of training. Grace should have had 4. Roderick should have had at least 12.)***

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Working with Proportions: Lesson 2.4

Since the focus of this lesson is on proportion, you will need to review the contents of the previous lesson, or start with Check Yourself on the preceding page.

Develop Your Understanding

Proportions

This lesson shows two different ways of writing proportions. While learners should be familiar with both, it's a good idea for them to choose the one they are most comfortable with and use it consistently until they are very familiar with this type of calculation.

Solving Proportions

Solving proportions requires learners to solve for an unknown algebraically. Solving for unknowns will be covered in more detail later in this module. For those who are intimidated by the word "algebra," the best approach may be to teach only the process as shown in the lesson without emphasizing the word. Later, they may be pleasantly surprised when they learn that they have already mastered some algebra!

Skill Builders

Learners work independently on this section. Have them check their work by using the Answer Key in the back of the book.

Check Yourself

The questions in this section test the learners' understanding of the meaning and uses of ratio and proportion problems. Review key points.

Lesson 2.4
Working with Proportions

Learning Objectives

To practice solving simple proportions
To set up and solve word problems using the concept of proportions
To practice calculating unit cost

In the previous lesson, you learned how to set up ratios and use them to solve problems. In this lesson, you will learn how to use two ratios to compare costs or amounts.

Vocabulary

proportion—a statement that two ratios are equivalent.
extremes—the two outside numbers in a proportion. In the proportion a:b = c:d, *a* and *d* are the extremes.
means—the two inside numbers in a proportion. In the proportion a:b = c:d, *b* and *c* are the means.
cross-product—in a proportion, the cross-product is the product of the extremes or the product of the means.

Develop Your Understanding

Proportions

Any statement that two ratios are equal or equivalent is called a *proportion*. When you found equivalent ratios in the last lesson, you were creating proportions.

$$\frac{2}{5} = \frac{4}{10}$$

Since the two ratios are equivalent, the above statement is a proportion.

> *An important property of proportions is that the* ***cross-products*** *of any proportion will always be equal.*

To find the cross-products in a proportion where the ratios are written as fractions:

1. Multiply the *numerator* of the first ratio by the *denominator* of the second ratio.
2. Multiply the *denominator* of the first ratio by the *numerator* of the second ratio.

Example 1

$$\frac{2}{5} = \frac{4}{10} \qquad \frac{2}{5} \times \frac{4}{10}$$

The first cross-product is the numerator of the first ratio (2) times the denominator of the second ratio (10).

$$2 \times 10 = 20$$

The second cross-product is the denominator of the first ratio (5) times the numerator of the second ratio (4).

$$5 \times 4 = 20$$

Note that the two cross-products are equal.

$$20 = 20$$

Another way to find the cross-products is to write the proportion using colons.

Example 2

The proportion $\frac{2}{5} = \frac{4}{10}$ can also be written:

$$2{:}5 = 4{:}10.$$

Here, the two numbers on the outside (2 and 10) are called the *extremes*, while the two numbers on the inside (5 and 4) are called the *means*. To find the cross-products:

1. Find the product of the two extremes.
2. Find the product of the two means.

Extremes: $2 \times 10 = 20$

Means: $4 \times 5 = 20$

Once again, the two cross-products are equal.

$20 = 20$

Now Try These

Find the cross-products for each proportion:

a. $\frac{1}{6} = \frac{2}{12}$ ***12***

b. $\frac{3}{11} = \frac{6}{22}$ ***66***

c. $\frac{7}{6} = \frac{21}{18}$ ***126***

d. 3:1 = 9:3 ***9***

e. 4:2 = 16:8 ***32***

Solving Proportions

Knowing that the cross-products of any proportion will be equal, you can find a missing number in a proportion. As long as you have one ratio and half of the other ratio, you can use cross-products to complete the proportion.

To solve a proportion:

1. Set the first ratio equal to the second.
2. Use a letter such as "N" (for "number") to take the place of an unknown number.
3. Find the cross-products of the ratios and set them up as an equation. To multiply a number by "N," simply write the number next to N. ($4 \times N = 4N$)
4. Divide both cross products by the number next to "N." You will have the value of "N" by itself. ($4N \div 4 = N$)

Example 3

Find an equivalent ratio for $\frac{2}{8}$ *that will have a denominator of 16.*

Steps 1 and 2: Set up the first ratio as equal to the second. Use N to represent the unknown number.

$$\frac{2}{8} = \frac{N}{16}$$

Step 3: Find the two cross-products:

(Multiply the first numerator by the second denominator.)

$2 \times 16 = 32$

(Multiply the first denominator by the second numerator.)

$8 \times N = 8N$

Write the product of 8 and N as "8N," which is a shorter way of writing "8 × N."

Now, write the two cross-products as equal:

$32 = 8N$

Step 4: Divide both of the cross-products by 8 to get the answer:

$32 = 8N$

$\frac{32}{8} = \frac{\cancel{8}N}{\cancel{8}}$ *The two 8s on the right cancel each other out.*

$4 = N$ *Substitute "4" for "N":* $\frac{2}{8} = \frac{4}{16}$

To check your answer, find the cross-products and make sure they are equal.

Now Try These

Use cross-products to solve for the unknown value in each proportion.

a. $\frac{7}{2} = \frac{N}{6}$ ***N = 21***

b. 1:1 = 32:N ***N = 32***

c. $\frac{15}{N} = \frac{30}{50}$ ***N = 25***

d. N:18 = 1:9 ***N = 2***

Word Problems with Proportions

Proportions can be used to compare amounts.

Example 4

Suppose your car gets 50 miles to 2 gallons of gas. You would like to know how many miles you could drive with 8 gallons of gas.

Steps 1 and 2: You know that your basic ratio of miles per gallon is 50 miles: 2 gallons. You would write this as the first ratio in your proportion:

50:2 =

Next, you would like to know how many miles you could drive with 8 gallons of gas. *Put the number of gallons in the same position in both ratios.* Since the number of gallons is the second number in the ratio you have already written, it belongs in the same place in the new ratio:

50:2 = N:8 (where N stands for the number of miles)

Step 3: Now, find the two cross-products by multiplying extremes and means.

Extremes: $50 \times 8 = 400$
Means: $2 \times N = 2N$

Set the first cross-products equal to the second:

$$400 = 2N$$

Step 4: Divide both cross-products by 2 to find the value of N:

$$400 \div 2 = 2N \div 2$$

$$200 = N$$

The value of N is 200. Therefore, *you could drive 200 miles on 8 gallons of gas.*

Now Try These

a. You need 120 square feet of office space for every 4 employees. How many square feet would you need for 5 employees?

150 square feet

b. Last Tuesday at the Perfect Printing Company, the ratio of perfect books produced to flawed books produced was 15:1. If the total number of flawed books was 7, how many perfect books did Perfect Printing produce last Tuesday?

105 perfect books

Finding the Best Buy

Have you noticed that many stores now show "unit prices" that help you compare the price per ounce (or pound, gallon, etc.) of certain products? You can use ratios and proportions to calculate price comparisons on your own:

1. Write ratios with price on the top and units on the bottom.
2. Divide the price by the number of units for each ratio.
3. The ratio with the smallest quotient is the one with the cheapest unit cost.

Example 5

You need to order more pencils for your department. You can get a box of 250 Zappo pencils for $12.75 or a box of 150 EZ Rite pencils for $7.05. Which box is a better buy?

Step 1: In each case, the "unit cost" you will need to compare is the price of *one* pencil. In order to do this, write two ratios of cost to number of pencils:

Zappos: $$\frac{\textit{total cost}}{\textit{\# of pencils}} = \frac{\$12.75}{250}$$

EZ Rites: $$\frac{\textit{total cost}}{\textit{\# of pencils}} = \frac{\$\ 7.05}{150}$$

Step 2: Solve each ratio by dividing the total number of pencils into the total cost:

Zappos: $12.75 ÷ 250 = $0.051

One of the Zappo pencils is 5.1 cents.

EZ Rites: $7.05 ÷ 150 = $0.047

One of the EZ Rite pencils is 4.7 cents.

Since one EZ Rite pencil is cheaper than one Zappo, EZ Rites are the better buy.

Skill Builders

Skill Practice

1. The cross-products of proportions must be equal. Mark an "X" next to any of the following proportions that are *incorrect*:

   a $\frac{1}{2} = \frac{85}{164}$ *X*

b. $\frac{6}{8} = \frac{18}{24}$

c. $\frac{3}{1} = \frac{225}{75}$

d. 12:2 = 96:8 ***X***

e. 1:17 = 34:2 ***X***

f. 17:1 = 34:2

Find the cross-products of the proportions in problems 2 through 5:

2. 25:1 = 250:10 ***250***

3. $\frac{6}{9} = \frac{2}{3}$ ***18***

4. 1.5:3 = 5:10 ***15***

5. $\frac{11}{3} = \frac{33}{9}$ ***99***

Solve for the missing value in problems 6 through 10:

6. 18:3 = 36:N ***N = 6***

7. $\frac{3}{13} = \frac{12}{N}$ ***N = 52***

8. 2.5:1 = 50:N **_N = 20_**

9. $\frac{12}{144} = \frac{N}{12}$ **_N = 1_**

10. $\frac{8}{N} = \frac{40}{25}$ **_N = 5_**

Word Problems

11. It takes you 3 days to complete a project. How many projects can you complete in 9 days?

    3 projects

12. You need 5 gallons of paint to paint two rooms. How many gallons of paint will you need to paint 13 rooms of the same size?

    32.5 gallons

13. If 2 inches on a map represent 150 miles, how many inches will represent 500 miles?

    6.67 inches or 6 $\frac{2}{3}$ inches

14. Denise needs to hire 7 temporary employees for each special project in her department. Right now, she has 53 temporary employees, and she needs to hire 3 more. How many special projects does Denise's department have?

8 projects

15. Blotto brand typing paper costs $1.80 for 75 sheets.

a. What is the cost of one sheet?

2.4 cents

b. Typo brand costs $2.34 for 90 sheets. Is it a better buy than Blotto?

No (2.6 cents per sheet)

16. Out of every $100 spent on employee benefits at Plink Industries, $54 goes toward health care. If Plink spent a total of $70,000 on benefits last year, how much of this total went toward health care?

$37,800

17. The Spendit Store has a special on canned yams. Normally, you can purchase a one-pound can of yams for $1.20. This week, you can buy a can of yams that weighs 2 pounds, 4 ounces for only $2.70. What is the difference in unit price? (16 ounces = 1 pound)

no difference in unit price

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Converting Fractions and Decimals into Percents: Lesson 2.5

This lesson focuses on renaming decimals as percents and fractions. Since place value concepts are important prerequisites, it would be beneficial to review. Copy the chart in Lesson 1.1, which shows place values, for learners to use as a reference.

Develop Your Understanding

Model all examples for the learners. You may need to work through more conversion problems if learners have difficulty with specific examples. Practice reading decimals. This skill is valuable in converting decimals to fractions. Percents are converted to decimals, decimals to fractions, and then the fraction is *reduced* or renamed in simplest form.

Write on Board
A fraction is said to be in simplest form when the numerator and denominator are prime.

Review this concept thoroughly.

Skill Builders

Learners works independently or with peer assistance. Have them check their work by using the Answer Key in the back of the book. Provide more conversion problems if learners need more practice.

Lesson 2.5
Converting Fractions and Decimals into Percents

Learning Objectives

To practice changing fractions and decimals into percents and back again

Solving problems with percents has many everyday applications in business, the workplace, and at home. To solve percent problems, you will need to convert fractions and decimals into percents and back again.

Vocabulary

percent—percent means "parts of a hundred."

For example, 15% is 15 parts out of 100 parts. A percent can be written in fraction, ratio, and decimal form. A percent is usually expressed by writing a percent sign (%). You can write:

$$15\% = 15 \text{ hundredths} = \frac{15}{100} = 0.15$$

Develop Your Understanding

Converting Percents to Decimals

When solving percent application problems, you will often need to convert a percent into a workable form. One of the easiest ways to do this is to change a percent to a decimal.

To convert a percent to a decimal, drop the percent sign and move the decimal point two places to the left. Remember that a whole number has an invisible decimal point on its right side.

Example 1

$45\% = \underset{\smile\smile}{45} = 0.45$

Example 2

$0.2\% = \underset{\smile\smile}{002} = 0.002$

Example 3

$100\% = 1\,\underset{\smile\smile}{00} = 1.$

Note that the zeros after the decimal point are no longer needed after converting 100% to one.

Now Try These

Convert these percents to decimals.

a. 28% ***0.28*** b. 0.6% ***0.006*** c. 200% ***2.0***

Converting Decimals to Percents

To convert a decimal back to a percent, move the decimal point two places to the right and add a percent sign.

Example 4

$0.56 = \underset{\smile}{5}\,\underset{\smile}{6} = 56\%$

Example 5

$0.05 = \underset{\smile}{0}\,\underset{\smile}{5} = 5\%$

Example 6

$1.2 = 1\,\underset{\smile}{2}\,\underset{\smile}{0} = 120\%$

Note that you must add a zero to show place value when changing 1.2 to 120%.

Now Try These

Convert these decimals to percents.

a. 0.36 = ***36%*** b. 0.09 = ***9%*** c. 4.3 = ***430%***

Converting Fractions to Percents

You will sometimes convert fractions to percents when solving percent problems. You can do this by converting the fraction to a decimal and then changing the decimal to a percent.

To convert a fraction to a decimal, divide the numerator by the denominator. Then convert the decimal quotient to a percent by moving the decimal point two places to the right and adding a percent sign.

Example 7

Convert $\frac{1}{4}$ to a percent by dividing the numerator by the denominator. You read the fraction as "one divided by four."

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{-8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Next, convert the decimal to a percent.

$$0.25 = \underset{\smile\smile}{25} = 25\%$$

Now Try These

Convert these fractions to percents.

a. $\frac{1}{5}$ = ***20%*** b. $\frac{3}{4}$ = ***75%*** c. $\frac{3}{8}$ = ***37.5%***

Converting Percents to Fractions

To convert a percent to a fraction, start by writing the percent as a decimal. Then convert the decimal to a fraction. Remember you can reduce the fraction to lowest terms by dividing the numerator and the denominator by the greatest common factor (GCF).

Example 8

$$72\% = 0.72 = \frac{72}{100} = \frac{18}{25}$$

In example 8 you would say, "seventy-two percent is equal to seventy-two hundredths (decimal form) is equal to seventy-two hundredths (fraction form) is equal to eighteen twenty-fifths (reduced fraction form)."

Example 9

$$4\% = 0.04 = \frac{4}{100} = \frac{1}{25}$$

Example 10

$$125\% = 1.25 = \frac{125}{100} = 1\,\frac{25}{100} = 1\,\frac{1}{4}$$

Example 11

$$2.5\% = 0.025 = \frac{25}{1000} = \frac{1}{40}$$

Note that the decimal 0.025 is read "25 thousandths," and the fraction must be written with a denominator of 1,000.

The number of zeros in the denominator of the fraction matches the number of decimal places in the decimal version of the number.

Now Try These

Convert these percents to fractions.

a. 95% = $\frac{95}{100} = \frac{19}{20}$

b. 8% = $\frac{8}{100} = \frac{2}{25}$

c. 215% = $2\,\frac{15}{100} = 2\,\frac{3}{20}$

d. 3.4% = $\frac{34}{1000} = \frac{17}{500}$

Skill Builders

Skill Practice

Convert these percents to decimals.

1. 35% = ***0.35***
2. 7% = ***0.07***
3. 105% = ***1.05***
4. 0.9% = ***0.009***
5. 1.5% = ***0.015***

Convert these decimals to percents.

6. 0.46 = ***46%***
7. 0.02 = ***2%***
8. 1.29 = ***129%***
9. 0.375 = ***37.5%***
10. 0.085 = ***8.5%***

Convert these fractions to percents using the two-step process you have learned.

11. $\frac{1}{2}$ = ***50%***
12. $\frac{4}{25}$ = ***16%***
13. $\frac{7}{10}$ = ***70%***
14. $\frac{1}{8}$ = ***12.5%***
15. $\frac{7}{8}$ = ***87.5%***

Convert these percents to fractions using the two-step process you have learned.

16. $20\% = \frac{20}{100} = \frac{1}{5}$

17. $8\% = \frac{8}{100} = \frac{2}{25}$

18. $175\% = 1\frac{75}{100} = 1\frac{3}{4}$

19. $9.8\% = \frac{98}{1000} = \frac{49}{500}$

20. $10.9\% = \frac{109}{1000}$

Word Problems

Solve these word problems by converting fractions, decimals, and percents. Convert fractions to lowest terms in your answers.

21. Della trains at the company gym 3 out of every 5 working days. What percent of her workweek does she train?

    60%

22. A papercutter's work order required that he add $\frac{3}{16}$ inch to job specifications. Change $\frac{3}{16}$ of an inch to a decimal measurement.

0.1875 inch

23. The label on Rodney's shirt says it is 80% cotton. What fraction of the shirt is cotton?

$\mathbf{\frac{4}{5}}$

24. If a microchip production plant inspects 1 out of every 3 chips during final inspection, what percent is inspected?

33% (rounded)

25. Eighty-eight percent (88%) of the employees at a plant had taken their vacation leave before December. What fraction of the employees had taken leave before December?

$\mathbf{\frac{88}{100} = \frac{22}{25}}$

26. A city's sales tax is $0.06 for every dollar. Write that tax rate as a percent.

6%

27. In a recent graphics order, 5 out of every 200 forms were found to be defective. What percent was defective?

2.5%

28. A company found that 0.42 of its employees had been with the company ten years or longer. What fraction of the employees had worked there ten years or more?

$$\frac{42}{100} = \frac{21}{50}$$

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Solving Percent Problems: Lesson 2.6

This lesson provides practice in solving word problems involving percents.

Develop Your Understanding

Read and model all examples for learners. You may need to work through additional problems if learners have difficulty with specific examples. The formula $\frac{P}{W} = \frac{R}{100}$ can be used to solve all three common types of percent problems. Help learners identify the values of P, W, and R.

P = part
W = whole
R = % rate

This also provides a review of cross-multiplication.

Skill Builders

Learners work independently or with peer assistance. Have them check their work by using the Answer Key in the back of the book.

Check Yourself

Follow the same procedure as in Skill Builders.

Lesson 2.6
Solving Percent Problems

Learning Objectives

To practice finding a percentage of a number
To practice finding the percentage one number is of another number
To practice finding a number when a percentage of it is given

Solving percent problems is one of the most valuable math tools you can develop. You can calculate interest on loans and bank accounts, and figure taxes and discounts. You can apply percentage calculations to manufacturing production and quality control measurements. In this lesson, you will solve the three kinds of percent problems.

Develop Your Understanding

In the previous lesson, you practiced converting fractions and decimals into percents and back again. Now you will apply these conversion skills to solve percent problems.

How to Look at Percent Problems

Any percent problem has three elements:

the *whole*, the *part*, and the *percent*.

For example, in the following statement

Twenty percent of 200 people is 40 people:

200 people is the *whole*, or the total amount;
40 people is the *part*, or the portion involved; and
20% is the *percent*.

Whenever you solve a percent problem, you will be given two of these elements, and will have to find the third. You will learn in detail how to solve problems in this lesson. The chart below is a handy reference you can keep as a reminder:

When you know...	**and...**	**In order to find...**	**You should:**
the whole	the percent	the part	*multiply* the whole by the percent (in decimal form)
the whole	the part	the percent	*divide* the part by the whole: $\frac{\text{part}}{\text{whole}}$
the part	the percent	the whole	*divide* the part by the percent (in decimal form): $\frac{\text{part}}{\text{percent}}$

This chart may be confusing at first. If so, don't try to understand it now. Refer back to the chart after you finish each major section of the lesson, and it will make more sense.

Finding the Percentage of a Number

Some problems give you a number and ask you to find a certain percentage of that number. To find the percentage of a number

1. follow the steps for changing a percent to a decimal;
2. multiply the decimal by the number.

*Note: You will know the **whole** and the **percent**; you are asked to find the **part**.*

Example 1: What is 15% of 60?

Step 1: Convert 15% to a decimal:

$$15\% = \underset{\curvearrowleft}{15} = 0.15$$

Step 2: Multiply 60 by the decimal:

$$\begin{array}{r} 60 \\ \underline{\times\ 0.15} \\ 300 \\ \underline{60} \\ 9.00 \end{array}$$

The product is 9.

Conclusion: 15% of 60 is 9. In this problem, 60 is the *whole*, 15% is the *percent*, and 9 is the *part*.

Note that "15% of 60" means 15% multiplied by 60. When you see the words *"percent of,"* it means that you should multiply.

Now Try These

a. What is 25% of 84?	b. What is 54% of 16?
21	***8.64***

Finding the Percentage One Number Is of Another Number

Other types of percent problems give you two numbers and ask you to find what percent the first number is of the second. To find the percent one number is of another

1. make a fraction with the *part* over the *whole*; write the *part* as the numerator and the *whole* as the denominator;
2. convert the fraction to a decimal by dividing;
3. change the decimal quotient to a percent.

Note: You will know the ***whole*** *and the* ***part****; you are asked to find the* ***percent****.*

Example 2: 3 is what percent of 4?

Step 1: Make a fraction with the *part* over the whole.

$$\frac{3}{4}$$

Step 2: Convert the fraction to a decimal by dividing.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{-28} \\ 20 \\ -\underline{20} \\ 00 \end{array}$$

Step 3: Change the decimal to a percent.

0.75 = 75%

Conclusion: Thus, 3 is 75% of 4. In this problem, 4 is the *whole*, 75% is the *percent*, and 3 is the *part*.

Now Try These

a. 4 is what percent of 5?

80%

b. 15 is what percent of 75?

20%

Finding a Number When a Percent of It Is Given

In the third type of percent problem, you are given the portion that represents a certain percentage of a number and asked to find the original number. To find a number when a percent of it is given10

1. convert the percent to a decimal;
2. divide the given number by the decimal.

Note: You will know the ***part*** *and the* ***percent****. You are asked to find the* ***whole****.*

Example 3: 34 is 40% of what number?

Step 1: Convert the percent to a decimal.

40% = 0.40 = 0.4

Step 2: Divide the given number by the decimal.

$0.\underset{\smile}{4}\overline{)34.\underset{\smile}{0}}$ is equivalent to

$$\begin{array}{r} 85 \\ 4\overline{)340} \\ \underline{-32} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Conclusion: Thus, 34 is 40% of 85. In this problem, 85 is the *whole*, 40% is the *percent*, and 34 is the *part*.

Note that in this problem, you knew that 34 was a part of an unknown number. Using division, you found that it was a part of 85.

Now Try These

a. 120 is 80% of what number?	b. 6 is 10% of what number?
150	***60***

Review the chart at the beginning of this lesson before you do the Skill Builders section.

Skill Builders

Skill Practice

Find the percent asked for in each problem.

1. What is 25% of 80?

20

2. What is 75% of 40?

30

3. What is 2.5% of 12?

0.3

Find the percent one number is of another number in each of these problems.

4. 3 is what percent of 5?

   60%

5. 28 is what percent of 40?

   70%

6. 1 is what percent of 3?

   33.3%

Find a number when a percent of it is given for each problem.

7. 4 is 10% of what number?

   40

8. 200 is 80% of what number?

   250

9. 18 is 40% of what number?

   45

Use your skills in calculating percents to solve the following manufacturing production problems. Units can represent any product, such as books or computer parts.

Find the number of units that passed inspection in each case below.

10. 98% of 50 units = ***49*** good units

11. 85% of 980 units = ***833*** good units

12. 99.9% of 1,000 units = **_999_** good units

Find the percentage of good units in each ratio below. The numerator is the number of good units, and the denominator is the total number of units.

13. $\frac{36}{48}$ = **_75_** % good units

14. $\frac{750}{800}$ = **_93.75_** % good units

15. $\frac{1950}{2000}$ = **_97.5_** % good units

Find the total number of units in each case below.

16. 24 is 60% of **_40_** total units

17. 100 is 80% of **_125_** total units

18. 14 is 87.5% of **_16_** total units

Word Problems

19. A $10,000 car is purchased with a 12% down payment. How much is the down payment?

    $1,200

20. You and a friend spend $22.20 dining at a restaurant. How much will you tip the waiter if you tip 15%?

    $3.33

21. Linda's take home pay after deductions is $1,230 per month. Her net income is 75% of her gross income. What is her total monthly income before taxes?

    $1,640

22. A company spends 2.5% of its $1,000,000 dollar budget on rent. How much does it spend on rent?

    $25,000

23. The job specifications tell you to cut the furnace temperature by 14%. The furnace is set at 959°F. How much will the temperature decrease? What will the new temperature be?

    134.26°F decrease
    824.74°F new temperature

Module 2 Review #1
Lessons 2.1-2.6

Before answering the following questions, you may wish to review the key concepts in Lessons 2.1 - 2.6. This may be done by reading over the lessons, following a review done by your instructor, or working in groups to discuss the key facts of each lesson.

After this, try to solve each of the following without looking back at the lessons.

1. List three key things you should do first when solving word problems.

   Define the question.
   Determine key information.
   Determine what mathematical operation to use.

2. If $\frac{2}{5}$ of 5,000 employees at XYI Company contribute to an IRA, how many total employees contribute to an IRA?

   2,000

3. The current workforce at the Dallas, Texas, plant numbers 6,000. It is projected to grow by 15% in the coming year. How many new employees will be hired or will transfer to the Dallas plant?

   900

4. The scrap rate for the Blue Line in January was 17%. The total number of computer boards produced was 15,690. How many boards were defective and had to be scrapped?

   2,667

5. Estimate the following department costs, rounding to the nearest dollar:

$52.99
$23.40
$12.50
$72.35
$29.19

$190

6. If the scrap rates were as follows for last week, what was the average?

Monday: 12
Tuesday: 32
Wednesday: 8
Thursday: 9
Friday: 11

14

7. Estimate the total of the following bill:

$12.29
22.29
31.89
16.38
15.99

$98

8. What is the average downtime (in days) for the drill machine if the following totals were turned in:

Week 1: down 2 out of 5 days
Week 2: down 0 out of 5 days
Week 3: down 3 out of 5 days
Week 4: down 1 out of 5 days

0.3 day

9. A recipe calls for $\frac{3}{4}$ cup of water to make $\frac{1}{2}$ jar of paste. How much water would you use to make 2 jars of paste?

3 cups

10. $\frac{15}{30} = \frac{3}{B}$ What is the value of B?

6

11. A chemical add formula calls for:

3 parts chemical A
2 parts chemical B
4 parts chemical C

If you must add 4 parts chemical B, how many parts of chemical A and chemical C do you add to maintain your chemical balance?

A: 6 parts
C: 8 parts

12. If you get 12 gallons of gas for $18, how much would you get for $15?

10 gallons

13. $\frac{N}{24} = \frac{3}{8}$ What is the value of N?

9

14. $\frac{6}{x} = \frac{20}{30}$ What is the value of x?

9

15. 8% of 280 is how many?

22.4

16. $\frac{3}{20}$ = what percent?

15%

17. Of 1,625 boards, 4% had defects. How many had defects?

65

18. If your department has a total budget of $8,000, and $1,200 is spent on supplies, what percent is spent on supplies?

15%

19. Convert each of the following to a percent:

$\frac{1}{2}$ = **50%**

$\frac{4}{5}$ = **80%**

20 = **2000%**

0.20 = **20%**

0.02 = **2%**

20. Convert the following to decimals:

40% = **0.4**
4% = **0.04**
400% = **4**
37.5% = **0.375**

21. Convert the following to fractions:

2% = $\mathbf{\frac{1}{50}}$

35% = $\mathbf{\frac{7}{20}}$

0.50 = $\mathbf{\frac{1}{2}}$

0.24 = $\mathbf{\frac{6}{25}}$

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Standard Measurement (Part 1): Lesson 2.7

This lesson is designed for the learner to become comfortable with standard units of measure and their uses.

Develop Your Understanding

Before you begin the lesson, you may want to look over the measurement tables with the learners. Point out the different types of measurement, e.g., liquid or solid capacity, length, weight, etc. It is critical that learners understand the use of conversion factors. Cover the four mathematical operations while introducing the idea of conversion factors (the conversion factors used to move up or down a scale of measurement). Have learners refer to the tables to see how the standard system of measurement works.

Skill Builders

The questions in the Skill Builders follow the lesson format. The first section is fairly straightforward computation. Learners work independently on this section. Have them check their work by using the Answer Key.

Lesson 2.7
Standard Measurement (Part 1)

Learning Objectives

To become familiar with standard units of measure and their uses
To practice adding and subtracting measurements
To practice converting between different units of measurement

Many types of measurement are used in the workplace. Different units are used to measure length, weight, volume or capacity (dry or liquid), temperature, time, and so on. Understanding these units of measure and their relationship to each other can enhance your ability to carry out your job, troubleshoot possible problems, and create solutions.

Vocabulary

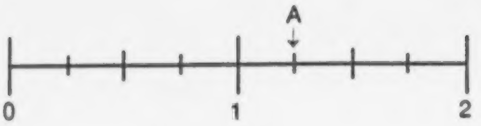
standard measurement system—the system of measurement used in the United States; includes feet, pounds, gallons, etc.
conversion—changing from one unit of measure to another; for example, changing from feet to inches.
conversion factor—the number or formula used to convert from one unit of measure to another.

Develop Your Understanding

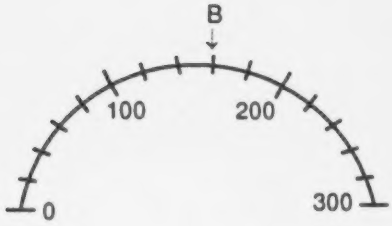
Reading Scales

Any type of measurement is done on a *scale*. A scale is a series of marks at certain distances from each other. Most scales are plotted along lines, but other shapes may also be used. You can learn to read any type of scale by following a few simple rules.

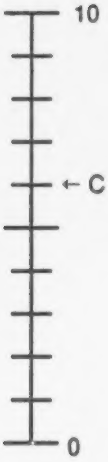
Examples of Scales



Scale 1



Scale 2



Scale 3

Each of the scales above has large markings that are labeled with numbers, as well as smaller markings in between that are not labeled. It is easy to read measurements at points that are labeled, but how about the ones in between?

Rules for Reading Scales

1. Find two labeled markings that are close together.
2. Subtract the lesser measurement from the greater one to find the distance between them.
3. Count the number of lines it takes to get from the first marking to the second.
4. Divide the distance between the labeled markings by the number of lines between them. This number shows how many units are represented by each line.
5. Add or subtract from the nearest labeled marking to find the measurement.

Example 1

Look at Scale 1, and find the measurement at point A.

Step 1: The two labeled markings nearest to point A are "1" and "2."

Step 2: Subtract: $2 - 1 = 1$. Therefore, 1 is the distance between the two markings.

Step 3: It takes 4 lines to get from marking 1 to marking 2.

Step 4: $1 \div 4 = \frac{1}{4}$. This means that every line represents $\frac{1}{4}$ of a unit.

Step 5: The nearest labeled marking is "1." Point A is one line past 1, so add:

$$1 + \frac{1}{4} = 1\frac{1}{4}.$$

The measurement at point A is $1\frac{1}{4}$ *units.*

Now Try These

Using the diagram "Examples of Scales," measure the number of units at:

a. point B on Scale 2: ***160 units***

b. point C on Scale 3: ***6 units***

Measurement Tables

The table on the next page shows standard units of measure and their relationships to one another. These relationships contain *conversion factors* you can use to go from one unit of measure to another. You may want to make a copy of this table, since you will often refer to it to solve measurement problems.

STANDARD MEASUREMENTS

Measure of Length

1 foot (ft.) = 12 inches (in.)
1 yard (yd.) = 3 feet = 36 inches
1 mile (m.) = 5,280 feet = 1,760 yards

Note: The symbol for inches is 5" = 5 inches
The symbol for feet is 5' = 5 feet

Liquid Capacity Measure

1 cup (c.) = 8 ounces (oz.)
1 pint (pt.) = 2 cups
1 quart (qt.) = 2 pints
1 gallon (gal.) = 4 quarts

Weight Measure

1 pound (lb.) = 16 ounces (oz.)
1 ton = 2,000 pounds

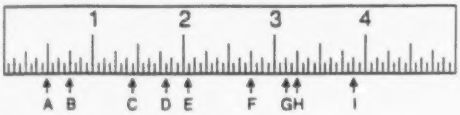
Reading a Standard Ruler

The standard ruler is one of the most basic types of scales used in the United States. The basic unit on a standard ruler is the inch. Inches are divided into sixteen fractional parts. These parts are called:

• half inches	*1 inch = 2 half inches*
• quarter inches	*1 inch = 4 quarter inches*
• eighths of an inch	*1 inch = 8 eighths of an inch*
• sixteenths of an inch	*1 inch = 16 sixteenths of an inch*

When reading a ruler, always reduce fractions of an inch to lowest terms, as you would do for any other fraction. For example, a measurement of $2\frac{8}{16}$ inches should be reduced to $2\frac{1}{2}$ inches.

Consider the following standard ruler:



What is the measurement marked at:

Point A? $\mathbf{\frac{1}{2}}$"

Point B? $\mathbf{\frac{6}{8}}$" **or** $\mathbf{\frac{3}{4}}$"

Point C? $\mathbf{1\frac{7}{16}}$"

Point D? $\mathbf{1\frac{13}{16}}$"

Follow the rules for reading scales to answer the above questions. Remember that half inches are marked by the longest lines in between inches; quarter inches are the next longest; and sixteenths of an inch are the shortest lines.

Finding the Distance Between Two Points

You can use two methods to find the distance between two points on a standard ruler:

Method 1: Count the number of units between the first point and the second.

For example, Point A lies on the $\frac{1}{2}$" mark. Point B is on the $\frac{6}{8}$" or $\frac{3}{4}$" mark. To find the distance in between them, look at the ruler and count the number of $\frac{1}{8}$" lines required to move from one to the other. There are two, so your answer would be $\frac{2}{8}$" or $\frac{1}{4}$" (reduced) between Point A and Point B.

Method 2: Use your knowledge of fractions and subtract.

$$\frac{6}{8} - \frac{1}{2} = \frac{6}{8} - \frac{4}{8} = \frac{2}{8}\text{"} \text{ or } \frac{1}{4}\text{"}$$

Try both ways to find the distance between Point B and Point C. $\mathbf{\frac{11}{16}}$"

Now Try These

Look back at the diagram of the standard ruler on the previous page, and answer these questions.

What measurement is marked at:

a. Point G? $\mathbf{3\frac{1}{8}}$"

b. Point E? $\mathbf{2\frac{1}{16}}$"

c. Point F? $\mathbf{2\frac{6}{8}}$" **or** $\mathbf{2\frac{3}{4}}$"

d. Point I? $\mathbf{3\frac{7}{8}}$"

What is the distance between:

e. Point B and Point D? $\mathbf{1\frac{1}{16}}$"

f. Point E and Point F? $\mathbf{\frac{11}{16}}$"

g. Point D and Point F? $\mathbf{\frac{15}{16}}$"

h. Point H and Point E? $\mathbf{1\frac{3}{16}}$"

Adding Measurements

Sometimes two measurements must be added to find a combined length. If you are adding inches, simply proceed as you would with any type of fractions. Find a common denominator, add, and reduce to lowest terms:

$$\frac{1}{2} + \frac{6}{8} = \frac{4}{8} + \frac{6}{8} = \frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4}$$

When adding feet and inches, however, it is best to use a *matrix*. Use rows and columns to keep different kinds of measurement separated:

Feet	Inches
2 ft.	10 in.
+3 ft.	5 in.
5 ft.	15 in.

As a rule, measurements should always be converted to the highest or largest type of unit possible: feet rather than inches, gallons rather than cups. In the matrix, adding 2 ft. 10 in. to 3 ft. 5 in. gives a sum of 5 ft. 15 in. Once you have a result, move any inches in excess of 12 to the "Feet" column. In this case, there are 15 inches:

15 in. - 12 in. = 3 in.
12 in. = 1 ft.

Therefore, 5 ft. 15 in. = 6 ft. 3 in., which is the final answer.

Now Try These

1. 3 ft. 6 in.
+6 ft. 11 in.
10 ft. 5 in.

2. 5 ft. 5 in.
+ 2 ft. 10 in.
8 ft. 3 in.

3. 9 ft. 4 in.
+3 ft. 6 in.
12 ft. 10 in.

Subtracting Measurements

Subtraction is done in a similar manner. Simply remember the basic scale of measurement. To subtract the following:

Feet	Inches
6 ft.	3 in.
-2 ft.	10 in.

Begin with the Inches column. Since 10 cannot be subtracted from 3, you will need to borrow one unit from the Feet column. BE CAREFUL!

> *Subtracting measurements is different from ordinary subtraction. In order to borrow from the next column, you must know how many of the smaller units are in one of the larger units.*

In this case, you know that *1 foot = 12 inches*. Therefore, when you borrow one from the Feet column, you must add **12** units to the Inches column:

Feet	Inches	=	Feet	Inches
6 ft.	3 in.	=	5 ft.	15 in.
-2 ft.	10 in.	=	-2 ft.	10 in.
			3 ft.	5 in.

The final answer is 3 ft. 5 in.

Now Try These

a. 5 ft. 5 in.
− 2 ft. 11 in.
2 ft. 6 in.

b. 12 ft. 1 in.
− 5 ft. 6 in.
6 ft. 7 in.

c. 4 ft. 7 in.
− 2 ft. 10 in.
1 ft. 9 in.

Multiplying or Dividing Measurements

Sometimes you may need to multiply a length to determine how much is needed to cover a larger distance, or to divide something into equal parts. When multiplying or dividing feet and inches, the rules are simple:

1. Multiply or divide the inches by the number.
2. Multiply or divide the feet by the number.
3. Change inches into feet if the answer has more than 12 inches.

Example 2: Multiply 2 ft. 2 in. by 7.

2 ft. × 7 = 14 ft.
2 in. × 7 = 14 in. 14 in. = 1 ft. 2 in.

The answer is 15 ft. 2 in.

Example 3: Divide 6 ft. 4 in. by 2.

6 ft. ÷ 2 = 3 ft.
4 in. ÷ 2 = 2 in.

The answer is 3 ft. 2 in.

Now Try These

a. 6 ft. 3 in.
× 5
31 ft. 3 in.

b. 3 ft. 10 in.
× 2
7 ft. 8 in.

c. 2 ft. 6 in.
× 6
15 ft.

d. 4 ft. 3 in. ÷ 3
1 ft. 5 in.

e. 7 ft. 4 in. ÷ 2
3 ft. 8 in.

Conversion of Units

These rules for adding, subtracting, multiplying, and dividing units of measure hold true regardless of what measurement unit you are using. The critical factor to know is the *conversion factor.* With this number you can convert to larger and smaller units of measure by multiplying or dividing.

Use the Standard Measurements conversion tables to convert measured amounts into different units. For example:

Liquid Capacity Measure

1 cup = 8 ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

How many ounces are in 2 pints?

You will need to work backwards in order to answer this question. The table shows the following information:

1 pint = 2 cups
1 cup = 8 ounces

If 1 pint contains 2 cups, then 2 pints contain 2 × 2 or 4 cups.
If 1 cup contains 8 ounces, then 4 cups contain 4 × 8 or 32 ounces.
Therefore, there are 32 ounces in 2 pints.

Now Try These

Use the measurement tables to answer these questions.

a. How many inches are in 7 feet?

84 inches

b. How many pints are in 3 gallons?

24 pints

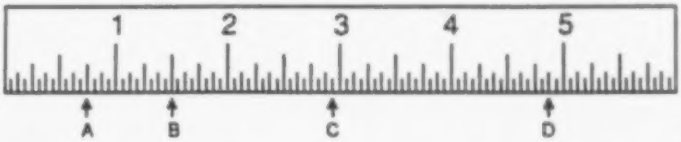
c. How many ounces are in 5 lb? (weight measure)

80 ounces

Skill Builders

Skill Practice

Use the following standard ruler to answer questions 1-5.



1. What is the measurement marked at Point A? $\frac{3}{4}$"

2. What is the measurement marked at Point B? $1\frac{1}{2}$"

3. What is the distance between Point C and Point D? $1\frac{15}{16}$"

4. What is the distance between Point A and Point D? $4\frac{1}{8}$"

5. Read the measurement at point B and the measurement at Point D. Add these measurements together. $6\frac{3}{8}$"

Solve the following.

6. 12 ft. 10 in. *(Convert the answer to yards, feet, and inches.)*
+ 8 ft. 9 in.

_7 yd. 7 in._

7. 16 ft. 3 in. *(Convert the answer to yards, feet, and inches.)*
−12 ft. 11 in.

_1 yd. 4 in._

8. Multiply:
6 ft. 3 in.
× 4

_25 ft._

9. Divide: 15 ft. 6 in. ÷ 2

_7 ft. 9 in._

10. 4 gallons 3 quarts
+ 2 gallons 3 quarts

7 gallons 2 quarts

11. 14 pounds 6 ounces
+ 8 pounds 10 ounces

23 pounds

12. 3 quarts 2 pints
– 2 quarts 3 pints

1 pint

Word Problems

13. Gerald needs to add some chemicals to the plating bath. He only has a 3-gallon container in which to mix them. If he mixes the following, will it fit into the container? *What is the total amount of the mixture?*

5 quarts Chemical A
3 pints Chemical B
6 cups Chemical C
2 quarts Chemical D
9 cups Chemical E

No, it is 1 cup too much.
3 gallons 1 cup total

14. The new crates will only hold 25 pounds each. If they are packed for shipment with the following items, will there be any weight limit problems?

50 ounces of shipping insulation
12 pounds of Kit XYZ
36 ounces paperwork
17 ounces plastic wrapping
68 ounces of Kit ABC

No, the total shipment is 22 pounds 11 ounces.

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Standard Measurement (Part 2): Lesson 2.8

This lesson is a continuation of Lesson 2.7. It includes a variety of measurement skills typically used in the workplace. These include area, perimeter, liquid volume, and time.

Develop Your Understanding

You may want to review the mathematical operations in Lesson 2.7 before you begin.

Skill Builders

The Skill Builders section mirrors the instructional format. It includes straight mathematical operation problems and more complicated word problems. Learners work independently on this section. Have them check their work by using the Answer Key in the back of the book.

Check Yourself

The section involves a time application problem that verifies student understanding of the 12-hour and 24-hour clocks, and the differences between them. Learners should be able to work the problem on their own, but some may need additional assistance.

Lesson 2.8
Standard Measurement (Part 2)

Learning Objectives

To practice measuring perimeter, area, liquid capacity, and time

In Lesson 2.7, "Standard Measurement (1)," you examined the mathematical logic necessary to work with measurement systems, and you also examined some of the measurement units of the standard system. In this lesson, you will examine other types of measurement, such as perimeter, area, liquid volume or capacity, and time.

Vocabulary

perimeter—the distance around the outside of an object
area—the amount of surface space inside an object, e.g., floor space in a room
volume or capacity—the total amount of space inside something; usually refers to the amount of something in a measuring container, such as one cup of water

Develop Your Understanding

Perimeter

An important measurement to be familiar with is *perimeter*, which is the distance around the outside of an object. Whether you are figuring the perimeter of circuit panels, buildings, or property, the formula is always the same: simply add the measurement of each side together. For example, if you have a six-sided property that you wish to put a fence around, you would simply add the length of each of the six sides to get the total perimeter.

Sometimes, you can take a shortcut to find the perimeter. For example:

> *A square, by definition, has 4 equal sides. Find the perimeter of any square by multiplying the length of one of the sides by 4.*
>
> *Opposite sides of a rectangle are also equal. Find the perimeter of any rectangle by multiplying the length by 2, multiplying the width by 2, and adding the products together.*

Now Try These

a. What would be the perimeter of a 5-sided building with measurements of 31 ft., 28 ft., 40 ft., 29 ft., and 30 ft.?

158 ft.

b. What would be the perimeter of a square in which one side is 5 inches?

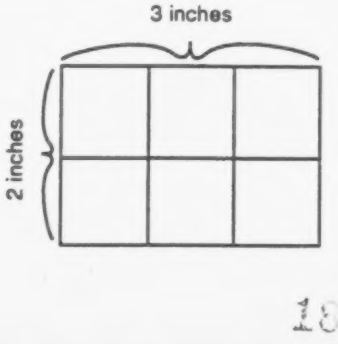
20 inches

c. What would be the perimeter of a rectangular panel that is 12 inches long and 11 inches wide?

46 inches

Area

Another important measurement is *area.* Area is the amount of flat surface space within an object. Measurements of area are expressed in *square units,* or units that have two dimensions: length and width. A square inch is a square that is 1 inch long and 1 inch wide. A square mile is an area that measures 1 mile on all four sides.



To find the area of any square or rectangle, multiply the length by the width. The product is expressed in square units: square inches (sq. in.), square feet (sq. ft.), etc. This formula ONLY works for squares and rectangles.

Example 1

Find the area of the rectangular part shown on the previous page.

The *length* of the part is 2 inches.
The *width* of the part is 3 inches.

2 in. x 3 in. = 6 sq. in.

The area of the part is 6 square inches.

Now Try These

a. What is the area of a panel that is 10 inches long and 9 inches wide?

90 square inches

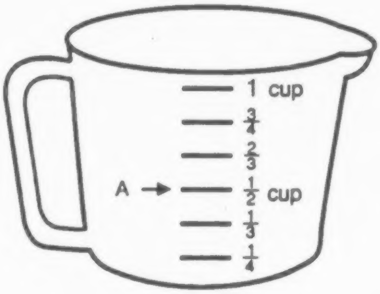
b. What is the area of a house that is 64 feet long and 25 feet wide?

1,600 square feet

Liquid Measurement

Forms of liquid measurement are often used at work. Sometimes this involves the standard system of cups, pints, quarts, etc., and sometimes metric measurement of volume or capacity is used. It is important to be familiar with both systems and to be comfortable with their use. You will have a chance to practice working with the metric system in the next lesson.

The standard system is the one most often used in cooking and other activities in the United States. For example, if you have the following measuring cup filled with water to the point marked by *A*, how much water would you have?



To find this measurement, read the marks on the measuring cup, and note that the water line comes to the $\frac{1}{2}$ cup mark. Therefore, there is $\frac{1}{2}$ cup of water in the cup. In the standard system, we measure whole units such as cups, pints, quarts, etc., or we measure some fraction of that whole.

Time

Depending on the system of time your place of employment uses, you may or may not be familiar with the 24-hour clock. This is the system commonly used by the military and many countries outside the United States.

Before working with the 24-hour clock, review how to make conversions and conduct mathematical operations with the 12-hour clock.

Example 2

How would you add the following times together to check your total time spent on a given project?

2 hours 20 minutes
3 hours 45 minutes
5 hours 35 minutes

Once again, you can use a matrix approach to solving this problem. To convert minutes to hours, use a conversion factor of 60 minutes to 1 hour.

Hours	Minutes
2 hrs.	20 min.
3 hrs.	45 min.
+ 5 hrs.	+ 35 min.
10 hrs.	100 min.

After adding the hours and minutes separately, check to see whether some of the minutes can be moved to the hours column. In this case, 100 minutes becomes 1 hour and 40 minutes:

Hours	Minutes
2 hrs.	20 min.
3 hrs.	45 min.
+ 5 hrs.	+ 35 min.
10 hrs.	100 min.
+ 1 hr.	- 60 min.
11 hrs.	40 min.

The total is 11 hours, 40 minutes.

The 24-hour Clock

The 24-hour clock labels the A.M. or morning hours with the numbers 1 through 12. Then, instead of starting over with 1 again, it continues to count upwards. This way 1 P.M. through 12 P.M. become the 13th through the 24th hours. Time on the 24-hour clock is expressed with four digits. The first two stand for the hour, and the second two stand for the minutes. For example, 0630 would be 6:30 A.M.

To calculate what a P.M. time would be on the 24-hour clock: add its hour number to 12. For example, 6 P.M. would be 12 + 6 = 18, or in 24-hour clock time, 1800 hours.

To calculate what a time on the 24-hour clock would be on the 12 hour clock: subtract 12 from the hour if it is greater than 12, insert a colon and add the letters "P.M." If the hour is 12 or less, insert a colon and add the letters "A.M."

Example 3

Express the following times in 12-hour clock time:

1520:
1230:
2400:

The first two digits represent the hour.

1520: The hour is greater than 12. 15 -12 = 3.
1230: The hour is not greater than 12. The numbers stay the same.
2400: The hour is greater than 12. 24 - 12 = 12.

Answers:

1520: 3:20 P.M.
1230: 12:30 P.M.
2400: 12:00 midnight

Note: On the 24-hour clock, 1200 is noon and 2400 is midnight.

Now Try These

a. Add the following times together, and find out the total length of time involved:

6 days	7 hours	
3 days	15 hours	
	12 hours	45 minutes
1 day	8 hours	50 minutes

11 days 19 hours 35 minutes

b. Convert the following times to the 12-hour clock system (remember to designate A.M. or P.M.):

0525: ***5:25 A.M.***
1730: ***5:30 P.M.***
2045: ***8:45 P.M.***

Skill Builders

1. Find the perimeter of the following:

a. A multisided building with these dimensions:

Side 1: 65 ft.
Side 2: 53 ft.
Side 3: 48 ft.
Side 4: 51 ft.
Side 5: 62 ft.

279 feet

b. A rectangular panel whose length is 12.3 inches and whose width is 11.5 inches.

47.6 inches

c. A square part with one side 8.6 inches long.

34.4 inches

2. Cindy needs to order boxes to ship some new panels. The panels are square-shaped with one side measuring 11 inches. The packing material will add 2 inches all the way around the panels. The best price on the boxes is for square ones that have an interior perimeter of 53 inches.

a. What is the total perimeter needed for the panels and packing material?

52 inches

b. Are these boxes big enough for the panels?

yes

3. Find the area of the following:

a. A square part with one side measuring 5 inches.

25 square inches

b. A rectangular building whose length is 25.3 ft. and whose width is 18.6 ft.

470.58 sq. ft.

4. ABC Computers has just gotten a contract for a new series of parts. The engineers have asked Linda to check their design to see if it meets with the needs of the company's machinery. The design is for a circuit panel that will have four parts on it. The panel is a rectangle that measures 11 inches by 12 inches. Each of the four parts is the same size. Each one is square-shaped with one side equal to 4.5 inches. There must be at least 2 free inches on the panel around each of the parts and between each of the parts to provide enough space for the machine clamps to hold each panel. *(Hint: Drawing a diagram will help you solve this problem.)*

   What is the total area of the panel?

   132 square inches

5. Add the following time spent on Project GHJ by Department Blue. What is the total time spent on it? (Convert to highest scale.)

8 days	12 hours	
3 days	9 hours	45 minutes
6 days	11 hours	55 minutes
13 days	7 hours	34 minutes

4 weeks 3 days 17 hours 14 minutes

6. Convert from 12-hour clock to 24-hour clock:

   a. 9:30 A.M. = ***0930***
   b. 5:30 P.M. = ***1730***
   c. 3:15 P.M. = ***1515***
   d. 11:45 A.M. = ***1145***

Convert from 24-hour clock to 12-hour clock (designate A.M. or P.M.):

e. 0520 = ***5:20 A.M.***
f. 1025 = ***10:25 A.M.***
g. 2230 = ***10:30 P.M.***
h. 1720 = ***5:20 P.M.***

Check Yourself

1. John spent the following times working on Project ABC:

Monday: 9:30 A.M. - 2:30 P.M.
Tuesday: 8:45 A.M. - 11:15 A.M.
Wednesday: 11:20 A.M. - 3:08 P.M.

a. How much total time did John spend on this project?

11 hours 18 minutes

b. John must record his work time for each project by hand on his time sheet. However, his time sheet calls for 24-hour clock time. Record John's times as they should be recorded on his time sheet:

Monday: ***0930–1430***

Tuesday: ***0845–1115***

Wednesday: ***1120–1508***

2. John records the following times on his time sheet for time spent on Project DEF:

   Monday: 0930 - 1345
   Tuesday: 1025 - 1250
   Wednesday: 0840 - 1410

   a. How much time did John actually spend on Project DEF?

      12 hours 10 minutes

   b. Convert to 12-hour clock time, and show the hours that John spent each day on Project DEF.

      Monday: ***9:30 A.M.–1:45 P.M.***

      Tuesday: ***10:25 A.M.–12:50 P.M.***

      Wednesday: ***8:40 A.M.–2:10 P.M.***

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Metric Measurement: Lesson 2.9

This lesson focuses on the basic principles of metric measurement and provides practice converting from one system to another.

Develop Your Understanding

As you teach this lesson, use as many teaching aids as possible: charts, rulers, etc.

Write on Board

Three ways to tell the difference between a standard ruler and a metric ruler:

- Metric ruler has 10 units.
  Standard ruler has 16 units.
- Metric ruler is labeled with *cm.*
  Standard ruler is labeled with *in.*
- Centimeters are visibly smaller than inches.

Skill Builders

It is suggested that learners work in groups or with a partner for "confidence" reinforcement.

Lesson 2.9
Metric Measurement

Learning Objectives

To learn the basic principles of metric measurement
To practice converting between standard and metric measurements

Although the standard system is still the principal method of measurement used in the United States, much of the rest of the world uses the metric system. As international trade increases, it will become very important to have a working knowledge of the metric system, and to be able to convert from one system to the other. Experience with the metric system will also be a great help to you if you ever travel outside the United States.

Vocabulary

metric measurement system—a system of measurement based on a scale of 10, used outside the United States and throughout most of the world; includes grams, meters, liters, etc.

Develop Your Understanding

The Metric System

Many people groan at the thought of learning a system of measurement other than the one they have used all their lives. You may feel that a new system would be more difficult than the one we have now. However, it could be argued that metric measurement as a system is more logical and easier to use than our standard system.

The metric system was developed by a group of scientists about 200 years ago. All of the measurement units in the metric system are based on a scale of 10, just like our number system. In other words, whenever you have *ten* of some kind of unit in the metric system, this is equal to *one* of the next larger unit. This is not true of the standard system, which has many different conversion factors for different types of measurement.

Each type of metric unit has a *prefix* and a *suffix*. The suffix tells what type of measurement it is: length, capacity, weight, etc. The prefix tells the size of the unit. Each type of measurement in the metric system has a basic unit which has no prefix.

METRIC SYSTEM TERMS

Prefix	**Meaning**
kilo-	basic unit times 1,000
hecto-	basic unit times 100
deka-	basic unit times 10
deci-	one-tenth (0.1) of basic unit
centi-	one-hundredth (0.01) of basic unit
milli-	one-thousandth (0.001) of basic unit

Suffix/Basic Unit	**Meaning**
meter	measure of length
gram	measure of weight
liter	measure of liquid capacity

Units of measurement are formed by combining a prefix with a suffix.

Example 1

A meter is the basic unit of length. What is a ***dekameter****?*

> Look on the chart and find the prefix "deka-." The meaning is "basic unit times 10."

A dekameter is 10 meters.

Now Try These

a. A liter is the basic unit of liquid capacity. What is a *milliliter?*

one-thousandth of a liter

b. A gram is the basic unit of weight. What is a *kilogram?*

1,000 grams

Conversions in the Metric System

Use the following tables to convert between units in the metric system. The tables also show abbreviations for each type of unit.

Measure of Length
Basic Unit: the meter (m)

10 millimeters (mm) = 1 centimeter (cm)
10 centimeters = 1 decimeter (dm)
10 decimeters = 1 meter (m)
10 meters = 1 dekameter (dam)
10 dekameters = 1 hectometer (hm)
10 hectometers = 1 kilometer (km)
1 kilometer = 1,000 meters (m)

Measure of Weight
Basic Unit: the gram (g)

10 milligrams (mg) = 1 centigram (cg)
10 centigrams = 1 decigram (dg)
10 decigrams = 1 gram (g)
10 grams = 1 dekagram (dag)
10 dekagrams = 1 hectogram (hg)
10 hectograms = 1 kilogram (kg)
1,000 kilograms = 1 metric ton (t)

Measure of Capacity
Basic Unit: the liter (l)

10 milliliters (ml) = 1 centiliter (cl)
10 centiliters = 1 deciliters (dl)
10 deciliters = 1 liter (l)
10 liters = 1 dekaliter (dal)
10 dekaliters = 1 hectoliter (hl)
10 hectoliters = 1 kiloliter (kl)
1,000 liters = 1 kiloliter

Because the metric system is based on 10, you can use a matrix approach to converting between units.

Looking at measures of length, for example, you might construct a matrix like this one:

kilometers km	hectometers hm	dekameters dam	meters m	decimeters dm	centimeters cm	millimeters mm

Fill in the matrix as follows:

Starting with the unit attached to a measurement, fill in one digit at a time, moving from *right to left.*

Example 2

Put these three measurements into the matrix: 91 mm, 312 cm, 423 m

After following the directions, your matrix should look like this:

kilometers km	hectometers hm	dekameters dam	meters m	decimeters dm	centimeters cm	millimeters mm
					9	1
			3	1	2	
	4	2	3			

The matrix is a very convenient way of converting units. A quick look at the matrix will tell you that:

91 mm is the same as 9 cm, 1 mm
312 cm is the same as 3 m, 1 dm, 2 cm
423 m is the same as 4 hm, 2 dam, 3 m

Now Try These

Use the matrix to express each measurement below in highest units.

kilometers km	hectometers hm	dekameters dam	meters m	decimeters dm	centimeters cm	millimeters mm

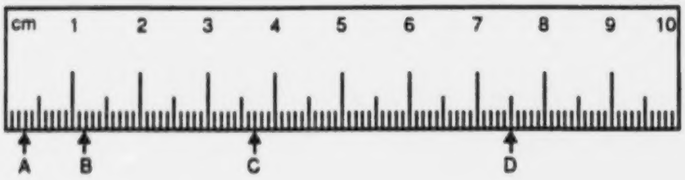
a. 221,400 millimeters =
2 hm   2 dam   1 m   4 dm

b. 35 decimeters =
3 m   5 dm

c. 6.5 centimeters =
(Remember that each unit is ***one tenth*** *of the unit to its left.)*
6 cm   5 mm

Using a Metric Ruler

The ruler shown below is a metric ruler. The shortest lines represent millimeters, and the tallest ones are centimeters. Note that "cm" is printed on the ruler to indicate that the lines labeled with numbers are centimeters. This ruler is one decimeter long. Ten decimeters equal one meter. The meter stick is a bit longer than our yardstick.



Example 3

How many millimeters are there between 12 mm and 2 mm?

To find the answer, all you have to do is subtract: 12 - 2 = 10.

There are 10 mm between 12 mm and 2 mm.

Now Try These

a. What measurement is marked on the ruler by: Point A? ***3 mm***
(answer in cm and mm) Point B? ***1 cm 2 mm***
Point C? ***3 cm 7 mm***

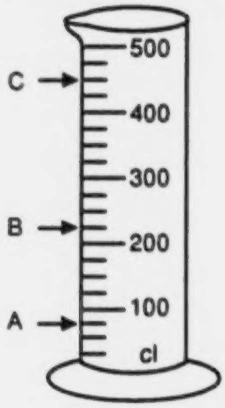
b. How far is it from Point A to Point D? ***7 cm 2 mm***

c. 7 mm + 10 mm = ***17*** mm = (convert) ***1*** cm ***7*** mm

Metric Capacity

The metric system uses liters as the basic measurement of volume or capacity. Since a liter is a fairly large measurement, roughly equivalent to one quart, most instruments for measuring capacity use milliliters, centiliters or deciliters as a basic unit. Remember to look for an abbreviation on the instrument to determine which units are labeled.

Study the metric cylinder below. Note that the labeled markings are *centiliters (cl).*



If the cylinder is filled with water up to Point A, how much water would you have?

(If you are not sure how to find the answer, go back to Lesson 2.7 on measurement).

The answer to the question on the previous page is 75 cl.
Since the distance between the labeled marks is 100 centiliters and it takes a jump of 4 lines to go from 100 to 200 cl, divide 100 by 4:

100 ÷ 4 = 25. Thus, each small line represents 25 cl.
Point A is three lines from the bottom: 25 × 3 = 75

Now Try These

a. What level is marked by Point B? ***225 cl***

b. What level is marked by Point C? ***450 cl***

c. How far is it between Point A and Point C? ***375 cl***

Converting Between Metric and Standard Systems

Do you remember that a *conversion factor* is the number that expresses the relationship between larger and smaller scales of measurement? Conversion factors can also be used to convert measurements from one system to another. In order to convert, you will need a table of conversion factors. These factors are easy to use, once you understand them. Because many of the factors are rounded, they are not always exact.

Metric to Standard Conversion Factors

Multiply:	**by:**	**to get:**
meters	39.37	inches
	3.28	feet
	1.09	yards
centimeters	0.39	inches
millimeters	0.04	inches
kilometers	0.62	miles
liters	1.06	liquid quarts
grams	0.04	ounces (weight)
kilograms	2.2	pounds
metric tons	2,204.62	pounds

Standard to Metric Conversion Factors

Multiply:	by:	to get:
inches	25.4	millimeters
feet	0.3	meters
yards	0.91	meters
miles	1.61	kilometers
liquid quarts	0.95	liters
dry quarts	1.1	liters
ounces (weight)	28.35	grams
pounds	0.45	kilograms
tons	0.91	metric tons

In order to use the conversion tables, you must first decide whether to convert from metric to standard or from standard to metric. This requires careful thinking.

Example 4

How many yards = 1 meter?

First, ask yourself: Should my *answer* be standard or metric?
In this case, you must answer in *yards*, a standard measurement.
Refer to this chart:

Answer in	**Use**
Standard units	Metric to Standard
Metric units	Standard to Metric

The chart tells you that, since you want an answer in standard units, you should use a metric to standard conversion:

multiply meters (1) by 1.09 to get the number of yards.

Answer: There are 1.09 yards in 1 meter.

How many kilometers = 1 mile?	***1.61 km = 1 mile***
How many ounces = 1 gram?	***0.04 ounce = 1 gram***

Now Try These

Convert each of the following:

a. 50 kilometers = ? miles

31.06 miles

b. 100 meters = ? yards

109 yards

c. 10 pounds = ? kilograms

4.5 kg

d. 25 grams = ? ounces

1 ounce

e. 16 ounces = ? grams

453.6 grams

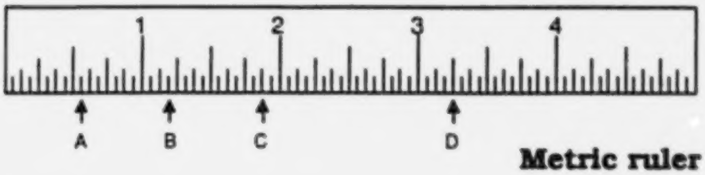
f. 3 liquid quarts = ? liters

2.85 liters

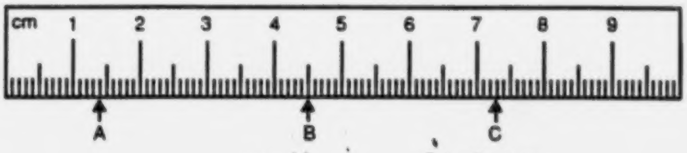
Skill Builders

Use the following illustrations to answer questions 1 and 2.

Standard ruler



Metric ruler



1. Find the ruler based on the standard system of measurement. What measurement is marked by:

   a. Point A? **$\frac{9}{16}$"**

   b. Point B? **$1\frac{3}{16}$"**

   c. Point C? **$1\frac{7}{8}$"**

   d. How far is it from Point A to Point C? **$1\frac{5}{16}$"**

2. Find the metric ruler.

   What point is marked by :

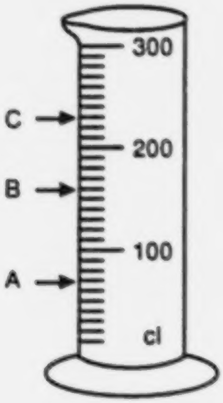
   a. Point A? ***1 cm 4 mm***

   b. Point B? ***4 cm 5 mm***

   c. Point C? ***7 cm 3 mm***

   d. How far is it from Point A to Point B? ***3 cm 1 mm***

3. Study the metric cylinder below, and answer the questions that follow.



   a. What level of measurement is marked by Point A? ***70 cl***
   Point B? ***160 cl***
   Point C? ***230 cl***

   b. How much liquid is contained between Point A and Point C? ***160 cl***

4. A roll of tape purchased in England has 2,000 meters of tape. How many *yards* of tape are in the roll?

   2,180 yards

5. Plink Industries packs items from its plant in Germany together with items made locally. The items from Germany come ready to be placed in the shipping crates. The contents of one crate are shown below. What is the combined weight of the items *in pounds?*

   Item 1: 30 kilograms
   Item 2: 3.5 pounds
   Item 3: 200 hectograms
   Item 4: 20 kilograms
   Item 5: 15 pounds
   Item 6: 105 ounces

   179 pounds 1 ounce

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Adding and Subtracting Signed Numbers:
Lesson 2.10

Before learners can develop skill in adding and subtracting signed numbers, they must understand the concept of signed numbers. This lesson is designed for mastering the concept of signed numbers, and overcoming the "fear" of algebra.

Develop Your Understanding

Read and discuss all rules for adding and subtracting signed numbers. Review all examples given in the Example section by drawing a number line on the blackboard or overhead.

Example

$(+5) + (-5) = 0$

Skill Builders

Research has shown that most learners need additional reinforcement in subtracting a negative number.

Learners work independently on this section. Have them check their work by using the Answer Key in the back of the book.

Check Yourself

Use this section as a group activity.

Tip

Same sign—same direction: add values.
Different sign—different direction: take the difference of the values and use the sign of the number with the greater absolute value.

Lesson 2.10
Adding and Subtracting Signed Numbers

Learning Objectives

To understand the concept of signed numbers
To develop skill in adding and subtracting signed numbers

Algebra is a powerful tool that simplifies problem solving, especially in technical fields. It is very useful for finding a missing piece of information when other pieces are known. When you begin to study algebra, you must learn to look at numbers in a new way. Before you begin this study, most of the numbers you use express values that are easy to see. You can show the meaning of the number *"4,"* for example, by counting *4 apples*. Some numbers, on the other hand, are *abstract* numbers that are difficult to show. Understanding how to use these abstract numbers in mathematical operations will enable you to solve new kinds of problems easily.

Vocabulary

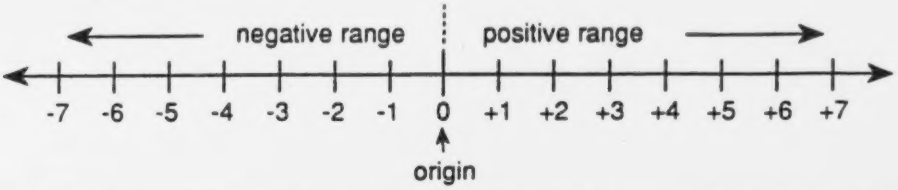
signed number—any number that carries a positive or negative sign.
integer—integers consist of whole numbers, their opposites, and zero.
absolute value—the distance an integer is from zero.

Develop Your Understanding

Integers and Signed Numbers

When you first studied math, you were probably taught that there is "no such thing as less than zero." But in real life, there are numbers that *are* "less than zero." For example, winter temperatures in northern climates often fall *below zero.* When a checking account is overdrawn, it is said to have a "*negative balance*"—this means a balance of less than zero.

The study of algebra begins with the study of *integers.* Integers include both *positive* and *negative* whole numbers. You are already familiar with positive whole numbers. Negative whole numbers are the opposites of positive whole numbers. The number line below shows some integers. Note that zero is also an integer.



The arrows at each end of the number line show that the line can go on forever in each direction. The point "0" on the line is called the origin.

All numbers to the **right** of the origin are called **positive numbers.** These numbers may be written *either* with a plus sign (+) in front or with no sign.

All numbers to the **left** of the origin are called **negative numbers.** These numbers must be written with a minus (-) sign in front.

The number line also represents many numbers that are not integers. Fractions and decimals as well as integers can be called **signed numbers.**

Example 1

The number (+7) can be read "seven" or "positive seven."
The number (-7) can be read "negative seven" or "the opposite of seven."

A number value with a sign is said to be the ***opposite*** *of the same number value with the opposite sign. For example, "(+3)" is the opposite of "(-3)."*

Absolute Value

The distance a signed number is from zero is called its *absolute value.* Distance can be measured from the origin to either the left or the right. The absolute value of zero is zero.

Example 2

The absolute value of both (-2) and (+2) is 2.
The absolute value of both (-100) and (+100) is 100.

Now Try These

Find the absolute value for problems a, b, and c.

a. The absolute value of (-9) is ***9***.

b. The absolute value of (+14) is ***14***.

c. The absolute value of $(-\frac{1}{2})$ is ____. $\frac{1}{2}$

d. What *two* signed numbers have an absolute value of 4? ***+4*** and ***-4***.

Adding Signed Numbers

Signed numbers can be added. Be sure to keep operational symbols (the "+" for addition) separate from the sign of a number by using parentheses. There are only two rules to remember for adding signed numbers. A different rule is used to add numbers with the same sign than to add numbers with different signs.

To Add Numbers with the Same Signs:
1. Find the absolute value of each number.
2. Add the absolute values together.
3. Keep the **original sign,** whether positive or negative.

Example 3

Add two positive numbers:

$(+5) + (+7) =$

1. The absolute values of (+5) and (+7) are 5 and 7.
2. Adding absolute values: $5 + 7 = 12$
3. The original sign of both numbers is (+). Therefore,

$(+5) + (+7) = (+12)$

Example 4

Add two negative numbers:

(-5) + (-7) =

1. The absolute values of (-5) and (-7) are 5 and 7.
2. Adding absolute values: 5 + 7 = 12
3. The original sign of both numbers is (-). Therefore,

(-5) + (-7) = (-12)

Now Try These

a. (+2) + (+6) = ***8***

b. (-4) + (-5) = ***-9***

c. (-3) + (-5) + (-200) = ***-208***

> *When adding two or more numbers, the order in which the numbers are added may be changed without changing the result. For example: 1 + 3 = 3 + 1.*

Sometimes you will add numbers with different signs. Remember this important rule:

To Add Numbers with Different Signs:

1. Find the absolute value of each number.
2. Subtract the smaller absolute value from the larger one.
3. Keep the sign of the number with the larger absolute value.

Example 5

Add a positive number to a negative number:

(+10) + (-4) =

1. The absolute values of (+10) and (-4) are 10 and 4.
2. Subtract 4, the smaller absolute value, from 10, the larger one.

   10 - 4 = 6

3. (+10) has the larger absolute value; its sign is (+). The answer will be positive:

   (+10) + (-4) = (+6)

Example 6

Add a negative number to a positive number:

(-9) + (+2) =

1. The absolute values of (-9) and (+2) are 9 and 2.
2. Subtract 2, the smaller absolute value, from 9, the larger one.

   9 - 2 = 7

3. (-9) has the larger absolute value; its sign is (-). The answer will be negative:

   (-9) + (+2) = (-7)

To Add a Series of Numbers:

1. Combine all positive values into one positive number.
2. Combine all negative values into one negative number.
3. Follow the rules for adding numbers with different signs.

Example 7

Add six numbers with different signs:

(+4) + (-3) + (-8) + (+3) + (-4) + (+8) =

1. Combine all positive numbers:

   (+4) + (+3) + (+8) = (+15)

2. Combine all negative numbers:

   (-3) + (-8) + (-4) = (-15)

3. Follow the rules for adding numbers with different signs:

   (+15) + (-15) =

Subtract absolute values: 15 - 15 = 0 (0 has no sign)

Therefore, (+4) + (-3) + (-8) + (+3) + (-4) + (+8) = 0

Adding any signed number to its opposite will result in an answer of zero.

Now Try These

a. 4 + 5 = ***9***

b. (+7) + (-8) = ***-1***

c. (9) + (5) = ***14***

d. (+6) + (-32) = ***-26***

e. (80) + (-21) + (7) + (-41) + (-1) = ***24***

Subtracting Signed Numbers

Signed numbers can also be subtracted. There is only one rule for subtraction.

To Subtract One Signed Number from Another:

Add the opposite of the number you are subtracting.

Example 8

Subtract a positive number from a negative number:

$(-5) - (+3) =$

Instead of subtracting (+3), you will add the opposite of (+3), which is (-3):

$(-5) - (+3) = (-5) + (-3)$

Now follow the steps for adding numbers with the same sign:

$(-5) + (-3) = (-8)$ *The answer is (-8).*

Example 9

Subtract a negative number from a negative number:

$(-10) - (-12) =$

Instead of subtracting (-12), you will add the opposite of (-12), which is (+12):

$(-10) - (-12) = (-10) + (+12) =$

Now follow the steps for adding numbers with different signs:

$(-10) + (+12) = (+2)$ *The answer is (+2).*

Now Try These

a. $5 - (-6) =$ ***11***

b. $14 - (+5) =$ ***9***

c. $45 - (+2) - (-7) =$ ***50***

Adding and Subtracting Signed Numbers: Summary

1. A positive number plus a positive number equals a positive number.
   (+) + (+) = (+)
2. A negative number plus a negative number equals a negative number.
   (-) + (-) = (-)
3. A positive number plus a negative number equals the difference between the two numbers, using the sign of the larger number.
   (+7) + (-2) = 7 - 2 = (+5)
4. To add several signed numbers, combine all positive and all negative numbers, and apply rule 3.
   (+2) + (-5) + (-7) + (+3) = (+5) + (-12) = (-7)
5. To subtract one signed number from another, add the opposite of the number to be subtracted.
   (-3) - (-2) = (-3) + (+2) = (-1)

Skill Builders

Skill Practice

Add (+5) to each of the following:

1. -8 ***-3***
2. +6 ***11***
3. -5 ***0***
4. -2 ***3***

Add (-34) to each of the following:

5. -17 ***-51***
6. +16 ***-18***
7. +23 ***-11***
8. +50 ***16***

Add these integers:

9. (-21) + (-16) = ***-37***

10. (+38) + (-19) = ***19***

11. (-55) + (-14) + (+28) + (+30) = ***-11***

12. (-102) + (+38) = ***-64***

Subtract (+7) from each of the following:

13. -64 ***-71***

14. +5 ***-2***

15. -31 ***-38***

16. +3425 ***3418***

Subtract (-31) from each of the following:

17. +45 ***76***

18. +2 ***33***

19. -701 ***-670***

20. +1 ***32***

Subtract these integers:

21. -6 -(-34) = ***28***

22. 4,567 - (-763) = ***5330***

Word Problems

23. Sue went to check the temperature of Solution A. It dropped to -12°F. How many degrees below freezing did the temperature fall? Remember, freezing = 32°F.

44°

☑

Check Yourself

Team A—DAILY PRODUCTION CHART—UR2 GRAPHICS

	Completed	*Pulled*
S. JONES	32	4
T. WELLS	29	2
J. SMITH	41	1

The above chart reflects the number of kits each employee completed and the number of kits Quality Control pulled. Team A's supervisor needs to turn in a report as to how many kits are left to be shipped after Quality Control pulled the damaged kits. Set this up as a mathematical problem with signed numbers, and solve it. *(Hint: Use a positive integer for each kit completed and a negative integer for each kit pulled.)*

$32 + 29 + 41 + (-4) + (-2) + (-1) = 95$

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Multiplying and Dividing Signed Numbers:
Lesson 2.11

This lesson focuses on two basic rules for multiplying and dividing signed numbers, and serves as a thorough review of all operations learned.

Develop Your Understanding

Discuss both rules and give other examples for division. This is a very easy concept for most learners and will require minimum instruction. Work all of the Now Try These on the board, or divide the group into two teams and allow them time to help each other.

Skill Builders

Learners work independently on this section. Have them check their work by using the Answer Key in the back of the book.

Check Yourself

Follow same procedure as stated in Skill Builders.

Lesson 2.11
Multiplying and Dividing Signed Numbers

Learning Objectives

To develop skills in multiplying and dividing signed numbers

This lesson will focus on learning and applying the rules for multiplying and dividing signed numbers. These rules must be mastered before solving algebraic equations in the workplace.

Develop Your Understanding

Multiplying Signed Numbers

To multiply two signed numbers, follow these rules:

same signs = positive answer   different signs = negative answer

Example 1

$4 \times 3 = 12$ positive × positive = positive
$-4 \times -3 = 12$ negative × negative = positive
$-4 \times 3 = -12$ negative × positive = negative
$4 \times -3 = -12$ positive × negative = negative

Note: A positive number can be written without the "+" sign.

Now Try These

Multiply the following, using the rules for multiplying signed numbers:

a. (-4) × (-8) = **32**

b. (-10) × (3) = **-30**

c. (4) × (-9) = **-36**

Dividing Signed Numbers

The basic rules for dividing signed numbers are the same as the rules for multiplying them:

same signs = positive answer   different signs = negative answer

Example 2

12 ÷ 4 = 3 positive ÷ positive = positive
-12 ÷ -4 = 3 negative ÷ negative = positive
12 ÷ -4 = -3 positive ÷ negative = negative
-12 ÷ 4 = -3 negative ÷ positive = negative

Now Try These

Divide the following, using the rules for dividing signed numbers:

a. (-49) ÷ (-7) = **7**

b. 68 ÷ 2 = **34**

c. (-40) + 5 = ***-8***

Multiplying Several Signed Numbers

To multiply several signed numbers, multiply two numbers at a time, and follow the rules for multiplication.

Example 3

$(-8) \times (-9) \times (-3) = -216$
$(-8) \times (-9) = 72 \times (-3) = -216$

Now Try These

a. (-4) × (-8) × (-6) × = ***-192***

b. 12 × 5 × (-80) = ***-4800***

Working with Other Signed Numbers

Fractions and decimals can be signed numbers, too. The same rules for multiplication and division apply to these numbers as to whole numbers. Be careful, however; remember that fractions and decimals have their own special rules for multiplication and division. Review those rules before working with signed fractions and decimals.

Example 4

$$\frac{-1}{2} \times \frac{1}{3} = \frac{-1}{6}$$

Example 5

$$(-12.6) \div (-2.1) = (+6)$$

$$-2.1\,\overline{)\,-12.6}$$

$$\begin{array}{r} +6 \\ -21\,\overline{)\,-126} \\ \underline{-126} \\ 0 \end{array}$$

Now Try These

a. $\frac{4}{5} \div \frac{-2}{3} = \mathbf{-1\ \frac{1}{5}}$

b. $(-5.2) \times (-10.4) =$ ***54.08***

Skill Builders

1. $(+6) \times (+100) =$ ***600***

2. $(-50) \times (-80) =$ ***4,000***

3. $(-2) \times (-3) \times (+2) = \mathbf{12}$

4. $(+2) \times (+1) \times (+6) \times (-1) = \mathbf{-12}$

5. $\frac{-2}{3} \times \frac{5}{8} = -\frac{\mathbf{5}}{\mathbf{12}}$

6. $7\frac{1}{3} \times (-6\frac{1}{3}) = \mathbf{-46\frac{4}{9}}$

7. $(-3) \times \frac{5}{6} = \mathbf{-2\frac{1}{2}}$

8. $$\begin{array}{r} -0.861 \\ \underline{\times\ -8.5} \\ \mathbf{7.3185} \end{array}$$

9. $$\begin{array}{r} 2.1 \\ \underline{\times\ 0.02} \\ \mathbf{0.042} \end{array}$$

10. $\left(\frac{-5}{7}\right) \times \left(\frac{-5}{8}\right) = \mathbf{\frac{25}{56}}$

11. (+10) ÷ (+100) = $\mathbf{\frac{1}{10}}$ ***or 0.1***

12. (+99) ÷ (-3) = ***-33***

13. (-56) ÷ (-2) = ***28***

14. 19.28 ÷ (-0.8) = ***-24.1***

15. 2.95 ÷ (-10) = ***-0.295***

16. $\frac{-1}{8} \div 1\frac{1}{8} = \mathbf{-\frac{1}{9}}$

17. -114 ÷ 6 = ***-19***

Module 2 Review #2
Lessons 2.7-2.11

After reviewing the key concepts in each lesson, solve each of the following problems. You may work all, or some, of the problems in each section. You will need extra paper.

Lessons 2.7, 2.8, and 2.9

1. Add the following, and convert to highest measurement:
   a. 3 yards 10 inches + 14 feet 6 inches + 22 inches ***8 yd. 2 ft. 2 in.***
   b. 5 gallons 25 ounces + 5 quarts 7 pints + 8 pints 12 cups
   9 gal 1 cup 1 oz.

2. Subtract the following, and convert.
   a. 12 feet - 16 inches ***10 ft. 8 in.***
   b. 3 quarts - 18 ounces ***2 qt. 14 oz. or 2 qt. + 1 cup + 6 oz.***
   c. 1 meter - 12 millimeters ***988 ml or 9 dm 8 cm 8 mm***
   d. 18 decimeters - 18 centimeters ***16 dm 2 cm or 162 cm***
   e. 1 deciliter - 10 milliliters ***9 cl or 90 ml***

3. Convert as instructed:
   a. 12 kilometers = ? miles ***7.44 miles***
   b. 12 miles = ? kilometers ***19.32 kilometers***
   c. 12 centimeters = ? inches ***4.68 inches***
   d. 50 meters = ? yards ***54.5 yards***
   e. 100 pounds = ? kilograms ***45 kilograms***
   f. 16 ounces = ? grams ***453.6 grams***

4. 16 feet 8 inches × 4 = ? ***22 yards 8 inches***

5. 3 yards 3 feet 6 inches × 5 = ? ***20 yards 2 feet 6 inches***

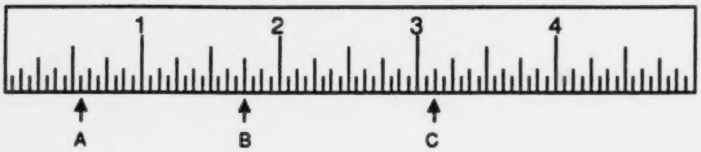
6. 11 feet 8 inches ÷ 4 = ? ***2 feet 11 inches***

7. 5 yards 4 feet 6 inches ÷ 2 = ? ***2 yards 3 feet 9 inches***

8. On the following ruler, what measurement is marked by Point A? $\mathbf{\frac{9}{16}}$"

Point B? $\mathbf{1\frac{3}{4}}$" Point C? $\mathbf{3\frac{1}{8}}$"

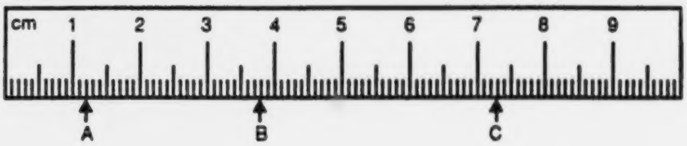
How far is it from Point A to Point B? $\mathbf{1\frac{3}{16}}$"
from Point B to Point C? $\mathbf{1\frac{3}{8}}$"
from Point A to Point C? $\mathbf{2\frac{9}{16}}$"



9. On the following ruler:

What measurement is marked by Point A? ***1.2 cm*** Point B? ***3.8 cm***
Point C? ***7.3 cm***

How far is it from Point A to Point B? **2.6 cm**
from Point B to Point C? **3.5 cm**



10. What is the perimeter of:

a. a square with one side equal to 8 inches? ***32 inches***

b. a rectangle with a height of 4 inches and a length of 12 inches? ***32 inches***

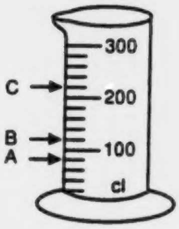
11. What is the area of the above two boxes? a. ***64 square inches***
b. ***48 square inches***

12. What measurement is marked on the following cylinder by:

Point A? ***80 cl***

Point B? ***120 cl***

Point C? ***220 cl***



13. Add the following:

3 days 22 hours + 5 days 16 hours 124 minutes
1 week 2 days 16 hours 4 minutes

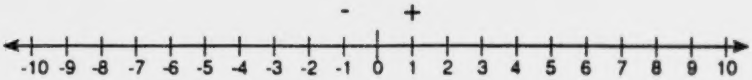
14. Convert the following:

0520: ***5:20 A.M.*** 1639: ***4:39 P.M.*** 2323: ***11:23 P.M.***

12:30 A.M.: ***0030 or 2430***
4:15 P.M.: ***1615***
6:20 P.M.: ***1820***

Lessons 2.10 and 2.11

15. Draw a number line that goes from –10 to +10. Draw a + and a – sign to show the direction for adding and subtracting positive numbers.



16. How would you write:

a. positive 10 plus positive 4? ***10 + 4 = 14***

b. negative 10 plus negative 4? ***-10 + (-4) = -14***

c. positive 10 minus positive 4? ***10 – 4 = 6***

d. negative 10 minus negative 4? ***-10 – (-4) = -6***

e. positive 10 plus negative 4? ***10 + (-4) = 6***

f. negative 10 plus positive 4? ***-10 + 4 = -6***

17. After writing out each of the above, solve them.

18. Solve each of the following:

a. 3 + 9 = ***12***

b. (–4) + (–5) = ***-9***

c. 2 + (–12) = ***-10***

d. 15 + (–5) = ***10***

e. 8 – 4 = ***4***

f. (–6) + (–5) = ***-11***

g. 4 – (–2) = ***6***

h. (–8) – (–6) = ***-2***

19. Solve each of the following:

a. 3 × 4 = ***12***

b. (–3) × 4 = ***-12***

c. (–3) × (–4) = ***12***

d. 3 × (–4) = ***-12***

e. 3 × 3 = ***9***

f. (–3) × (–3) = ***9***

g. 9 ÷ 3 = ***3***

h. (–9) ÷ (–3) = ***3***

i. (–9) ÷ 3 = ***-3***

j. 9 ÷ (–3) = ***-3***

k. 15 ÷ (–5) = ***-3***

l. (–20) ÷ (–2) = ***10***

m. 25 ÷ 5 = ***5***

n. (–30) ÷ 6 = ***-5***

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Algebraic Equations and Inequalities: Lesson 2.12

This lesson is intended to give learners a broad overview of the concepts of equations and inequalities.

Develop Your Understanding

Equations

You may want to introduce the symbol " $\neq$ " which means "is not equal to." Learners can draw a slash mark through the " = " sign to show that an equation is not true.

Order of Operations

Some learners may want to know why these rules are necessary. Demonstrate that simply following the symbols without regard to order of operations produces a different answer. The Order of Operations Rule was developed as a method for consistent interpretation, much like grammatical rules.

Parentheses

Point out that in some cases parentheses are used to set off a number and its sign, as in (-4) or 6 - (-2). Learners have seen examples of this in previous lessons.

Variables

Give learners ample practice in substituting values for variables. Some may need to be reminded that an unknown value is **not** equal to zero.

Inequalities

This is intended to be a basic introduction and does not include the "greater than" and "less than" symbols (>, <). You may wish to introduce them to these symbols once they have firmly grasped the concepts in this lesson.

Converting Words to Equations and Inequalities

Emphasize the importance of writing out known and unknown values *in words* before trying to solve the problem, as well as the importance of showing **all** work.

Lesson 2.12
Algebraic Equations and Inequalities

Learning Objectives

To become familiar with some of the basic symbols used in algebraic equations and inequalities
To memorize and apply the Order of Operations Rule for solving equations
To practice setting up equations and inequalities based on the information given in word problems

The ability to solve equations and inequalities is one of the basic skills you will need in order to work algebra problems. First, you will need to review what equations and inequalities are, and how they can be used to find the answers to problems that you might encounter at work.

Vocabulary

equation—a mathematical sentence containing an "=" sign which states that two expressions are equal.
variable—a symbol, usually a letter, that holds the place of a number when the value of the number is unknown. Variables can be either capital letters (A, B, C, X, Y, etc.) or lowercase letters (a, b, c, x, y, etc.).
inequality—a mathematical sentence that states that two expressions are *not* equal

Develop Your Understanding

Equations

An equation is a mathematical statement containing the symbol "=." When you see the "=" symbol, it means that the numbers or expressions on one side of the symbol have the same value as those on the other side, even if they do not look the same.

Some examples of equations are:

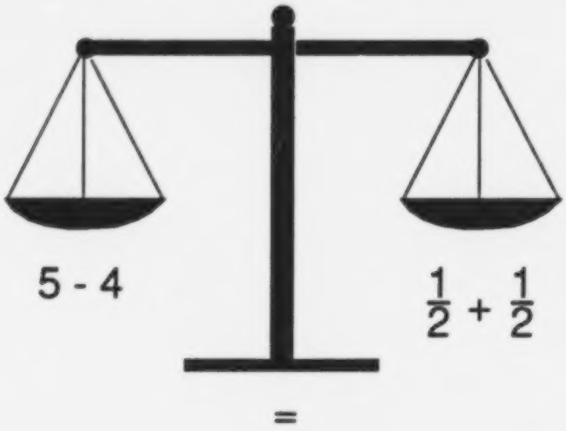
$$2 + 3 = 5$$

$$5 - 4 = \frac{1}{2} + \frac{1}{2}$$

$$3 \times 6 = 9 \times 2$$

$$-4 \times -2 = 8 + 1$$

An equation is like a scale that is perfectly balanced:



You can check an equation to make sure it is true. For example:

$$5 - 4 = \frac{1}{2} + \frac{1}{2}$$

$$5 - 4 = 1$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

Since 1 = 1, this equation is true.

In order for an equation to remain in balance, anything that is done to the left side must also be done to the right side. If, for example, 200 is added to both sides of an equation, it will remain true:

$$5 - 4 = \frac{1}{2} + \frac{1}{2}$$

$$200 + 5 - 4 = \frac{1}{2} + \frac{1}{2} + 200$$

Now Try These

Check each equation to make sure it is true.

a. $1 - \frac{1}{2} = 0.25 + 0.25$

true

b. $3 \times 9 = 100 - 75$

not true

c. $45 \div 5 = -6 + 15$

true

Solving Equations: The Order of Operations

You are probably used to seeing open-ended equations where you are given the left side of the equation only and must solve it to find the right side. This is simple when there is only one operation involved:

$$46 - 28 =$$

To solve this equation, you would simply subtract 28 from 46 to get 18.

What happens when there is more than one operation in an equation? How would you go about solving it? There is a simple rule for this, called the *Order of Operations Rule.*

Order of Operations Rule

1. First, do anything contained inside parentheses.
2. Perform multiplication and division operations, moving from left to right.
3. Finally, add and subtract, moving from left to right.

You will have a chance to work with parentheses in the next section. For the following example, apply only rules 2 and 3.

Example 1

Solve the equation:

$12 - 3 \times 2 =$

Applying the Order of Operations Rule, you would first multiply or divide from left to right:

$3 \times 2 = 6$

The equation now looks like this:

$12 - 6 =$

Now, add or subtract from left to right:

$12 - 6 = 6$

Therefore, $12 - 3 \times 2 = 6$

Now Try These

a. $6 - 7 \times 3 + 15 =$ ***0***

b. $-10 + 5 \times 6 + 4 = \boldsymbol{-8}$

c. $85 - \frac{2}{3} \times 3 = \boldsymbol{83}$

The Uses of Parentheses

Some equations contain parentheses. Parentheses are used in a variety of ways:

- Mathematical expressions (including operations) that are inside parentheses should be simplified *first* when solving an equation:

  $9 \times (3 - 6) = 9 \times (-3) = -27$

- When a number appears immediately in front of a set of parentheses, the expression inside the parentheses is multiplied by that number after it is simplified:

  $9(2 + 4) = 9(6) = 9 \times 6 = 54$

- When two sets of parentheses appear side by side, the expression inside the first one should be multiplied by the expression in the second one after both are simplified:

  $(-2)(-3) = -2 \times -3 = 6$

  $(7 - 12)(4 + 2) = (-5)(2) = -10$

Remember, always simplify all expressions contained in parentheses before you go on to solve the equation.

Example 2

How would you solve the following equation? Would the answer be the same if you moved from left to right without following the Order of Operations Rule?

$4 - 2(6 - 2) \div 2 =$

First, to solve the equation properly, simplify everything inside parentheses:

$4 - 2(6 - 2) \div 2 = 4 - 2(4) \div 2$

Next, multiply and divide from left to right:

$4 - 2(4) \div 2 = 4 - 8 \div 2 = 4 - 4$

Finally, add and subtract from left to right:

$4 - 4 = 0$

The solution is 0. Would you get the same answer if you just did the problem from left to right without worrying about parentheses or the type of operation? Try just moving from left to right, ignoring parentheses and the Order of Operations Rule. Your answer will be something like 5. Be sure to follow the order of operations to arrive at the correct answer.

Now Try These

Solve each equation. Be sure to write down all the steps.

a. $2(10 - 12) =$ ***-4***

b. $3 - (-8 - 6) =$ ***17***

c. $3(-4 \times 3) + 2(6 - 7) =$ ***-38***

d. $100 - 7(-3 - 4 + 2) =$ ***135***

Variables

Sometimes an equation contains a *variable*. Variables are letters of the alphabet, used to take the place of a number when we do not know its value. Some examples of equations containing variables are:

$2 + a = 7$	"a" is the variable
$R(12) = 120$	"R" is the variable
$\frac{3 - n}{2} = 5$	"n" is the variable
$6(b) = b + 10$	"b" is the variable

(A variable may appear more than once in the same equation.)

When a variable is written next to a number, this means it is multiplied by that number:

$5r$ means "5 times r"

$16j$ means "16 times j"

$6q = 18$ means "6 times q equals 18"

Once you know the value of a variable, you can substitute the value into the equation to replace the variable.

Example 3

Solve the equation $7z = ?$ where $z = 4$. Since you know that $z = 4$, replace z with 4:

$7(4) = ?$

When a number replaces a variable in this way, use parentheses to show the difference between 7(4) and 74.

$7(4) = 28$

Example 4

When you have an *open-ended* equation with a variable to solve, the variable can be any number you choose:

$3 - x = ?$

For the value of x, let's select 3, 15, and -290.

where $x = 3$, $3 - x = 3 - 3 = 0$

where $x = 15$, $3 - x = 3 - 15 = -12$

where $x = -290$, $3 - x = 3 - (-290) = 3 + 290 = 293$

Example 5

When an equation with one variable is *closed* (both sides of the equation are shown), there will be only one solution:

$16T - 25 = 7$

$3v - 18 = v$

In each of these cases, there is only one possible value for T or v. You will learn how to solve problems of this type in Lesson 2.13.

Now Try These

For each open-ended equation below, solve for the values of each variable indicated.

a. $2d - 1 = ?$ where $d = -5$ and $d = 30$

-11 ***59***

b. $3g = ?$ where $g = -1$ and $g = 3$

-3 ***9***

c. $22 - 2f = ?$ where $f = 15$ and $f = -11$

-8 ***44***

Inequalities

We have explored *equations*, which are mathematical statements that two expressions have the same value. An *inequality* is a statement that shows that two expressions have different values and compares them.

The symbols ">" and "<" are used in inequalities in place of the "=" sign.

Example 6

When the closed end of the inequality sign points to the right, the sign is read "is greater than."

When you see:
$5 > 3$
Read:
"Five is greater than three."

When the closed end of the inequality sign points to the left, the sign is read "is less than."

When you see:
$8 < 10$
Read:
"Eight is less than ten."

Remember, the open end of the inequality sign points to the larger value; the closed end points to the smaller value.

Now Try These

Insert the symbol "<", ">" or "=" in the space between each set of values:

a. $24 \underline{\quad < \quad} 30$

b. $1.254 \underline{\quad > \quad} 1.2$

c. $\frac{1}{2} \underline{\quad < \quad} 2$

d. $\frac{1}{2} \underline{\quad = \quad} 0.5$

e. $300 \underline{\quad > \quad} 296$

f. $3 \underline{\quad < \quad} \frac{19}{6}$

Inequalities with Variables

Sometimes a variable may appear in an inequality.

$x > 4$ may be read "x is greater than 4"

We don't know the exact value of x, but we know it could be any number larger than 4, including a fraction or decimal value.

Sometimes, an inequality can show a value that is between two other numbers:

$3 < x < 17$ may be read "x is greater than 3 and less than 17"

$55 > y > 22$ may be read "y is less than 55 and greater than 22"

If we know that the value of y is between 55 and 22, then $y = 20$ could not possibly be a true statement.

Now Try These

For each example, look at the inequality. Then write "possible" or "impossible" for each value of the variable given. The first one is done for you:

	Inequality	**Value**	**Is the value possible?**
a.	$13 < d$	$d = 11$	*impossible (11 is not greater than 13)*
b.	$45 > M$	$M = 0$	***possible***
c.	$-3 < p$	$p = -1$	***possible***
d.	$100 < z < 101$	$z = 100.734$	***possible***
e.	$7 < L < 9$	$L = 6$	***impossible***

Converting Words to Equations and Inequalities

When you solve word problems, the most challenging task is translating what is told to you in words into mathematical equations or inequalities that you can then solve. Below are several examples that should help you.

Example 7

How would you set up an equation to solve this problem?

Two-thirds of the panels produced on Thursday needed to be rewired. The number of panels needing rewiring was 512. How many panels were produced on Thursday in all?

You know how many panels needed rewiring, but you don't know how many were produced.

First, *assign a variable to the unknown number:*

let N = total number of panels produced

Now, use the information you already have to set up a relationship between the variable and known values:

Two-thirds of the panels is equal to 512.

$$\frac{2}{3} N = 512$$

You could then go on to solve the equation mathematically and find the answer to the problem. You will study this procedure in the lessons that follow.

Example 8

When the 25 employees at a small Plink Industries distribution office gave a staff party, they all shared the cost of the food. The organizers of the party promised that no one would have to spend more than $6.00. Fifteen of the employees, "The High Rollers," each spent twice as much as the other ten ("The Budgeteers"). Each High Roller spent the same amount as every other High Roller; each Budgeteer spent the same amount as every other Budgeteer. The total spent came to $100. How much did each of the ***Budgeteers*** *spend? Set up the equation(s) or inequalities you would need to solve this problem.*

First, what is the value you need to know? It is the amount of money spent by each *Budgeteer*. Let's call this "B":

B = amount (in dollars) spent by each Budgeteer.

You know that nobody spent *more than* $6.00; therefore, B must be *less than* $6.00:

$$B < 6$$

Each High Roller spent twice as much as each Budgeteer. Therefore, the amount spent by a High Roller is the amount spent by a Budgeteer multiplied by 2. Since the amount spent by a Budgeteer is "B," then:

$2B$ = amount (in dollars) spent by each High Roller.

The total spent was exactly $100. This is the total of ten times "B," or the amount each Budgeteer spent, plus fifteen times "2B," or the amount each High Roller spent:

$$10B + 15(2B) = 100$$

AND, $B < 6$

You now have all the information in the problem in mathematical form. In the next two lessons, you will solve problems of this type.

Now Try These

Write the information given in each problem in the form of equations or inequalities, choosing your own variables.

a. Molly ordered three times as many pairs of safety glasses for her area in January as she did in February. The total number of pairs ordered for the two months was 24. How many pairs did she order in February?

$3s + s = 24$

s = number of pairs ordered in February

b. Donald has boxed and shipped a total of 1,569 books in the last three weeks. How many books has he averaged per week?

A = average number per week
$3A = 1{,}569$

Skill Builders

1. Is this equation true? $30 - 18 = 24 \times \frac{1}{2}$ ***yes***

2. Is this equation true? $1{,}800 \times 0.01 = 3 \times 5$ ***no***

3. $144 - (2 \times 8) \div 4 + 18 =$ ***158***

4. $25 - (45 \times \frac{1}{5}) + (10 \div 2) =$ ***21***

5. $(-5) \times (-3) - (14) =$ ***1***

6. $(2)(4)(\frac{1}{8}) =$ ***1***

7. $3 - 2(-6 + 4) \div 2 =$ ***5***

8. $-(12 - 6)(3 \times 4) =$ ***-72***

9. $88 + (-5 - 11) \times (-3 + 5) =$ ***56***

For problems 10-12, substitute each value given for the variable into the equation and solve it.

10. $3x + 17 = ?$

a. where $x = 3$

26

b. where $x = -3$

8

11. $y \div 2 + 114 = ?$

a. where $y = 100$

164

b. where $y = 1$

114.5 or $\boldsymbol{114\frac{1}{2}}$

12. $6A - A = ?$

a. where $A = 11$

55

b. where $A = -1$

-5

13. If $x < 25$, is it possible that $x = 0.002$?

yes

14. Fill in ">" or "<" for each blank.

a. $21 \underline{\ <\ } 3(8)$

b. $4x \underline{\ >\ } 92$ if $x = 26$

c. $(\frac{1}{2})(\frac{1}{4}) \underline{\ <\ } \frac{1}{4}$

d. $0.925 \underline{\ >\ } 0.9205$

15. If $1 < w < 2$, what is one possible value for w?

Answers will vary; must be a fraction or decimal between 1 and 2.

For problems 16-20, use the information given to set up equations or inequalities using variables. DO NOT try to solve the problems!

16. Frieda bought and ate a candy bar on Monday. On Tuesday, she bought two candy bars and ate them both. On Wednesday, she bought three candy bars and only ate two; on Thursday she ate the one that was left. On Friday, Frieda bought and ate four candy bars. Her total spent on candy bars for the week was $6.40. If each of the bars was the same price, what is the price of one candy bar?

p = price of one candy bar

$p + 2p + 3p + 4p = \$6.40$

17. Zelda drank more than thirty 6-packs of Rhubarb Delight soda pop last year. Is it possible that she drank only 174 cans of this delicious beverage?

c = number of cans Zelda drank

if $c > 6\ (30)$ could $c = 174$?

18. Phil sees a lot of movies. He saw an average of 4 per week during a certain period of time. During that time, he saw a total of 52 movies. How many weeks were included in that period?

w = number of weeks

$4w = 52$

19. When the budget for Plink Industries was decided last year, the Marketing Department received five times as much money as the Special Projects Department. The manager of the Special Projects Department complained, and got an additional $80,000 added to her annual budget. As a result, the two departments wound up with the same amount of money. How much did Special Projects get before the additional money was given?

m = amount of money Special Projects got

$5m = m + \$80{,}000$

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Addition and Subtraction, Solving for an Unknown: Lesson 2.13

This lesson builds on the concepts taught in previous lessons with the focus on solving for an unknown using addition and subtraction.

Develop Your Understanding

You may want to review the key facts about algebraic equations before beginning instruction. Try to avoid math jargon, as learners may have a high degree of anxiety about algebra before they even start.

When walking through the lesson, you may wish to explain that inverse operations are opposite to each other.

Skill Builders

The Skill Builders include a number of simple equations to solve. They also include some problems with multiple mathematical operations. Walk through an example. This shows that you can combine numbers before adding or subtracting on both sides of the equation.

Some of the Skill Builders also include negative numbers.

Learners work independently on this section. Have them check their work by using the Answer Key in the back of the book.

Check Yourself

This section includes two word problems that require the learner to set up and solve equations. Although learners can reason out the correct answer, encourage them to solve for the unknown using addition or subtraction.

Lesson 2.13

Addition and Subtraction, Solving for an Unknown

Learning Objectives

To understand and practice the method used to find the value of an unknown number using addition and subtraction

In everyday life and in the workplace, there may be times when you will need to know the value of an unknown number. Many times you may figure it out by reasoning through the problem or situation. In this lesson, you will examine a very simple method to use whenever you need to find the value of an unknown number.

Vocabulary

A, B, X, Y or any letter used in a mathematical problem—the letter simply stands for an unknown value, another way of saying "I don't know." In the previous lesson, these letters were called *variables*.

substitution—replacing a variable with the number that it stands for.

Develop Your Understanding

In the last lesson, you learned about algebraic equations, what they stand for, and basically how to work with them. In this lesson you will apply this knowledge as you find the value of unknown variables in addition and subtraction problems.

Remember that any time you see a variable in a math problem, the letter simply means "I don't know" and stands for an unknown number. Variables are useful because they remind you that there is a particular number that belongs there. As soon as you know what the number is, you can substitute it for the variable.

As you learned in the last lesson, an algebraic equation is simply a mathematical sentence with an equal sign in the middle. It means that one side equals the other side. Because the two sides are equal, you must keep them that way when working with the problem. Therefore, whatever you do to one side, you must also do to the other side.

Addition

In order to solve an algebraic equation, you will need to isolate the variable on one side of the equal sign. When the variable is by itself on one side of the equal sign, the other side of the equation will tell you the value of the variable.

Keep in mind that equations must remain balanced. Therefore, you must do the same thing to ***both*** *sides of the equation.*

Steps in Solving for an Unknown: Using Addition

1. Look at the equation and find what operation is used.

$$b - 3 = 5$$

In the above equation, 3 is subtracted from some number to get 5. You could probably work this out by reasoning through it, but it is useful to practice the steps to solve equations with an easy problem like this.

2. Isolate the variable by applying the **opposite** of whatever math operation is being done.

$$b - 3 + 3 = 5 + 3$$

The opposite of subtraction is addition. Therefore, **add** 3 to **both** sides of the equation. When (+3) is added to (-3), the opposites cancel each other out to zero:

$$b - 3 + 3 = 5 + 3 =$$
$$b - 0 = 5 + 3$$

3. Simplify both sides of the equation.

$$b = 5 + 3 \qquad b = 8$$

Zero subtracted from any number leaves that number. The value of the variable "b" is 8.

4. Check your answer by substituting it into the original equation and solving.

   $8 - 3 = 5$

   Yes, the equation is true. "$b = 8$" is the correct solution.

Now Try These

Use the steps demonstrated above to find the value of the following equations. Remember to get the letter by itself by doing the opposite mathematical operation of that done on the letter side, and applying this to both sides of the equation.

a. $x - 22 = 14$

36

b. $V - 5 = 95$

100

c. $d - 12 = 18$

30

Subtraction

In the above examples, you solved equations by addition. The same rules hold for subtraction. If you examine an algebraic equation and find that you are adding a number to the letter side, perform the opposite operation, which is subtraction, to get the letter by itself.

Steps in Solving for an Unknown: Using Subtraction

1. Look at the equation and find what operation is used.

   $F + 6 = 20$

   In the above equation, 6 is added to some number to get 20.

2. Isolate the variable by applying the **opposite** of whatever math operation is being done.

   To find out what that number is, simply subtract 6 from **both** sides of the equation.

   $F + 6 - 6 = 20 - 6 \qquad F + 0 = 20 - 6$

3. Simplify both sides of the equation.

   $F + 0 = 20 - 6 \qquad F = 14$

4. Check your answer by substituting it into the original equation and solving.

   $14 + 6 = 20$

   Therefore, 14 is the correct value for "F."

Now Try These

Solve the following equations.

a. $J + 10 = 75$

65

b. $m + 8 = 23$

15

c. $P + 4 = 48$

44

Word Problems

In everyday life, equations to find an unknown value do not compose themselves. You must apply the facts of a situation and set up an equation on your own, and then solve for the unknown number.

To do this, apply the skills you have learned in earlier lessons. First, identify what the key information is and then what mathematical operation should be applied.

Example

The Blue Team finished a certain number of panels during week 1, 82 panels in week 2, and 28 panels in week 3. They also had to scrap 29 panels. This means that the total yield was 125 panels. How many panels did they produce in week 1? Solve this by setting up an equation as follows:

Let X = the number of panels finished during week 1

$$X + 82 + 28 - 29 = 125$$

Begin by performing the mathematical operations as shown and combining 82, 28, and -29 into one number. After that, you want to get the X by itself, so you will do the opposite operation to any numbers on the letter side of the equation.

$$X + 82 + 28 - 29 = X + 81$$

Now, substitute this simplified number into the equation:

$$X + 81 = 125$$

Isolate the variable by performing the opposite operation:

$$X + 81 - 81 = 125 - 81$$

$$X = 44$$

Put 44 into the original equation to check the answer:

$$44 + 82 + 28 - 29 = 125$$

Therefore, they produced 44 panels during week 1.

Now Try These

Our production this week included 78 layers for one job, 32 for another, and an unknown number for a third. We do know that our final numbers included 12 scrapped (discarded) layers and a final figure of 140 layers produced after the 12 were scrapped. How many layers were produced in the third job? Set up your equation and solve for the unknown.

$$\boldsymbol{78 + 32 + X - 12 = 140}$$

$$\boldsymbol{X = 42}$$

Skill Builders

Solve for the unknown variable.

1. $a - 10 = 25$

   35

2. $B + 12 = 12$

   0

3. $n - 25 = 60$

   85

4. $K + 15 = 75$

   60

5. $j - 23 = 35$

   58

6. $V + 47 = 22$

   -25

7. $p - 23 = -12$

   11

8. $C - 5 + 6 = 15$

14

9. $d + 12 - 8 = 22$

18

10. $E - 6 - 7 = 225$

238

11. $f + 18 + 23 - 14 = 62$

35

12. $R + 22 - 7 + 4 = 10$

-9

13. $c + 105 - 23 = -40$

-122

14. $T + 19 + 58 = -20$

-97

15. $y + 23 - 40 - 9 = 0$

26

Check Yourself

Set up and solve each of the following equations.

1. We ordered the following number of safety smocks during November: 23, 46, 12, 74. When we finished the month, 48 had been discarded, and some replacements ordered. Our inventory showed 125 on hand at the end of the month. How many replacement safety smocks were ordered?

$$\begin{aligned} 23 + 46 + 12 + 74 - 48 + X &= 125 \\ X &= 18 \end{aligned}$$

2. Our production numbers for January were 5% higher than those in December. Our December production was 595 layers completed. If we had to discard a certain number for defects and our final January production was 590, how many defective layers were produced? (Round your answer to the nearest whole number.)

$$\begin{aligned} 595 + (595)(5\%) - X &= 590 \\ \text{or } 595 + (595)(.05) - X &= 590 \\ X &= 35 \end{aligned}$$

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Multiplication and Division, Solving for an Unknown:
Lesson 2.14

This lesson provides instruction and feedback on solving for an unknown using multiplication and division.

Develop Your Understanding

Read and discuss all rules and examples. You may need to remind learners of some algebraic properties.

Write on Board

Multiplicative Inverse

$$y \times \frac{1}{y} = \frac{1}{y} \times y$$

$$y \bullet \frac{1}{y} = \frac{1}{y} \bullet y$$

Multiplicative Identity

$$y \bullet \frac{1}{y} = \frac{1}{y} \bullet y = 1$$

Definition of Division

$$x \div y = x \bullet \frac{1}{y}$$

Multiplicative Property of Equality

if $x = y$, then $x \bullet c = y \bullet c$

Division Property of Equality

if $x = y$ and $c \neq 0$, then $\frac{x}{c} = \frac{y}{c}$

Emphasize isolating the variable or the unknown.

Another way to make learners feel more comfortable is to show how they learned algebra in grades 1–4. The only difference is in the terminology.

Skill Builders

Learners work independently on this section. Have them check their work by using the Answer Key.

Check Yourself

Follow the same procedures as in Skill Builders.

Lesson 2.14
Multiplication and Division, Solving for an Unknown

Learning Objectives

To understand and practice the method used to find the value of an unknown number using multiplication and division

When an equation contains an unknown or variable, algebra is used to find the value of that unknown. In this lesson, you will develop skill in solving for an unknown in equations using multiplication and division, and then apply that skill to some common problems found in the workplace.

Develop Your Understanding

Multiplication

In the previous lesson, you practiced solving for an unknown in addition and subtraction equations by using the opposite operation. Multiplication and division are opposites in the same way that addition and subtraction oppose each other. To solve for an unknown in a multiplication equation, use division.

Steps in Solving for an Unknown: Using Division

1. Look at the equation and find what operation is being done.

$$3x = 12$$

Ask yourself, "what has been done to the variable?" Here, it has been multiplied by 3.

2. Isolate the variable by applying the **opposite** of whatever math operation is being done.

The opposite of multiplication is division.

$\frac{3x}{3} = \frac{12}{3}$ Divide each side by 3.

$\frac{\cancel{3}}{\cancel{3}}x = \frac{12}{3}$ The 2 "3's" in the fraction on the left cancel each other out.

3. Simplify both sides of the equation.

$x = \frac{12}{3}$ $12 \div 3 = 4$

$x = 4$

The value of the variable "x" is 4.

4. Check your answer by substituting it into the original equation and solving.

$3(4) = 12$

The equation is true, so $x = 4$ is the correct answer.

Now Try These

Solve for the value of each variable.

a. $4b = 8$

b = 2

b. $7c = 63$

c = 9

Division

To solve for an unknown in a division equation, use multiplication.

Steps in Solving for an Unknown: Using Multiplication

1. Look at the equation and find what operation is used.

   $\frac{x}{2} = 6$

   Ask yourself, "what is being done to the variable?" Here, the variable "x" is divided by 2. Remember that the fraction bar indicates division. In other words, $\frac{x}{2}$ can be read "x divided by 2."

2. Isolate the variable by applying the **opposite** of whatever math operation is being done.

   The opposite of division is multiplication.

   $2 \times \frac{x}{2} = 6 \times 2$ Multiply both sides by 2.

   $\frac{\cancel{2}x}{\cancel{2}} = 6 \times 2$ $6 \times 2 = 12$

3. Simplify both sides of the equation.

   $\frac{\cancel{2}}{\cancel{2}}x = 12$ The 2s in the fraction on the left cancel each other out.

   $x = 12$

   The value of the variable "x" is 12.

4. Check your answer by substituting into the original equation and solving.

   $\frac{12}{2} = 6$

   The equation is true, so $x = 12$ is the correct answer.

Now Try These

Solve for the value of each variable.

a. $G \div 6 = 5$

$\boldsymbol{G = 30}$

b. $\frac{m}{9} = 3$

$\boldsymbol{m = 27}$

Special Types of Problems

Proportions

In an earlier lesson, you learned that an equation in which two ratios are equal is called a *proportion.* You can cross-multiply and solve for the unknown as follows:

$\frac{7}{5} = \frac{91}{g}$ Find the cross-products, and set them as equal.

$$\frac{7}{5} \quad \times \quad \frac{91}{g}$$

$7g = 5\ (91)$

$7g = 455$

$\frac{\cancel{7}g}{\cancel{7}} = \frac{455}{7}$ Divide both sides by 7 to isolate the *g*.

$g = \frac{455}{7}$ Solve for *g*.

$g = 65$

Fractions of a Variable

Just as a variable can be *part* of a fraction, a variable can also be multiplied by a fraction. In this case, treat the variable as a whole number. Any whole number can be written as a fraction using a denominator of 1.

$\frac{1}{3}$ y means "$\frac{1}{3}$ times the variable y"

$$\frac{1}{3}y = \frac{1}{3} \times \frac{y}{1} = \frac{1 \times y}{3 \times 1} = \frac{y}{3}$$

To solve an equation containing a fraction of a variable, simplify first:

$$\frac{1}{3}y = 5$$

$$\frac{1}{3}y = \frac{y}{3}$$

$$\frac{y}{3} = 5$$

Now solve as a proportion:

$$\frac{y}{3} = \frac{5}{1} \qquad y \times 1 = 3 \times 5$$

$$y = 15$$

The value of the variable "y" is 15. Substitute into the original equation to check:

$$\frac{1}{3}(15) = 5 \qquad \frac{1}{3} \times \frac{15}{1} = \frac{1 \times 15}{3 \times 1} = \frac{15}{3} = 5$$

The equation is true.

Now Try These

Solve for the value of each variable.

a. $\frac{8}{x} = 4$

$x = 2$

b. $\frac{1}{3}z = 9$

$z = 27$

Skill Builders

Solve and check each multiplication equation.

1. $9y = 126$

$y = 14$

2. $4a = 52$

$a = 13$

3. $\frac{1}{2}y = 10$

y = 20

4. $91 = 13x$

x = 7

5. $\frac{1}{5}x = 2$

x = 10

6. $5x = 15.5$

x = 3.1

7. $2w = 32$

w = 16

Solve and check each division equation.

8. $\frac{a}{8} = 16$

$a = 128$

9. $\frac{w}{7} = 21$

$w = 147$

10. $\frac{x}{2} = 10$

$x = 20$

11. $\frac{d}{25} = 5$

$d = 125$

12. $\frac{x}{9} = -9$

$x = -81$

13. $\frac{c}{16} = \frac{1}{2}$

c = 8

14. $\frac{k}{112} = 2$

k = 224

Use all problem-solving skills, and solve the following:

15. Sandra spends 22% of her gross salary to buy mutual funds through her company. This 22% equals $440.00 a month. What is Sandra's monthly gross salary?

$2,000.00

16. The artist at Goodway Graphics was told to draw a map for their delivery truck driver using 2 inches to represent 150 miles. How many inches will represent 500 miles?

$6\frac{2}{3}$ or 6.67 inches

17. If 5 times a number is equal to 25, what is the number?

5

18. A thin metal disc is 3 millimeters thick. If 120 of these discs are to be stacked in the yellow room at XI, how many centimeters thick will the stack be?

36 centimeters

19. If a number is divided by 7, and the quotient is 7, what is the number?

49

20. Patrick was told to find the voltage of an electrical current which has 12 amperes of electrical current in the circuit. Let V = the number of volts. Using $V = \frac{10}{3} \times 12$, solve for V.

40 volts

Check Yourself

Solve the following equations:

1. $0.2x = 10$

   $x = 50$

2. $2x + 4 = 10$

   $x = 3$

3. $2x - 0.4 = 10$

   $x = 5.2$

4. $\frac{x}{2} = 10$

   $x = 20$

5. $\frac{1}{2}x = 10$

   $x = 20$

6. $x + \frac{1}{2} = 10$

   $x = 9\frac{1}{2}$

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Working with Formulas: Lesson 2.15

Using formulas to solve certain math problems is introduced as a guideline for solving certain problems in the workplace.

Develop Your Understanding

Explain to learners that the formula $I = prt$ means $I = p \times r \times t$. Variables placed directly next to each other will be multiplied together. Also, the formulas for area and for distance can be written as $A = lw$ and $D = rt$. You can omit the multiplication sign.

Emphasize writing the formula every time that you solve a problem. Explain that errors are often made by learners who want to skip this step.

Order of Operations

In this module we do not have any formulas with exponents. If you bring in formulas that include exponents, add another step in the order of operations.

1. Do everything inside the parentheses first.
2. Do all multiplication and division in order going from left to right.
3. Do all addition and subtraction in order from left to right.

Some learners may be confused in thinking that they must do multiplication before division or addition before subtraction. Clarify that a division operation may come before a multiplication operation as they work the problem from left to right. If it does, then they should perform the division first. The same is true when a subtraction operation comes before an addition operation.

Skill Builders

A variety of formulas are included in this section. You may want to have learners bring in formulas from their work areas that may be more specific to their jobs.

Learners work independently on this section. Have them check their work by using the Answer Key.

Check Yourself

Learners may be using formulas without realizing it. Emphasize that a formula is a shorthand set of instructions that states the solution to a problem and can be used over and over again to perform a specific task. Learners can list concrete uses of formulas as in figuring gas mileage, following instructions for recipes, and mixing substances.

Lesson 2.15
Working with Formulas

Learning Objectives

To solve math problems using formulas

A formula is a general guideline for solving similar types of problems. As you work with formulas, you will need to apply most of your math skills. The key to using formulas successfully is substituting accurate numbers for variables.

Vocabulary

formula—an equation that can include letters (variables), symbols, and numbers. A formula is a shorthand set of instructions that states the solution to a problem.

Develop Your Understanding

Look at these examples of formulas:

$A = L \times W$	Area = Length × Width
$I = prt$	Interest = principal × rate × time
$D = r \times t$	Distance = rate × time

When you work with formulas, you will be given the formulas and the definitions of those formulas. You will also be given the values (numbers) of all of the letters (variables) except one. You must solve the problem to find the unknown value.

Example 1

Find the area of a floor when the length is 20 feet and the width is 15 feet.

A = L × W	Write the formula.
A = 20 feet × 15 feet	Substitute the values.
$\begin{array}{r} 20 \\ \times 15 \\ \hline 100 \\ 20 \\ \hline 300 \end{array}$	Solve for the unknown.

A = 300 square feet

You were given the values for length and width. You solved the problem to find the unknown value. You found that the area was 300 square feet. When solving problems using formulas, follow these steps.

1. Write the formula. *(Do not skip this step!)*
2. Substitute the values for the letters.
3. Solve for the unknown value.

Now Try These

a. Find the area when the length is 6 feet and the width is 3 feet.

A = L × W

A = 18 square feet

b. Find the gas mileage (in miles per gallon) when you traveled 300 miles and used 20 gallons of gasoline.

$$G = \frac{m}{g}$$

G = 15 miles per gallon

$$\text{Miles per gallon} = \frac{\text{miles you drove}}{\text{gallons of gas you used}}$$

You can even make your own formula.

Example 2

Write a formula for total earnings for the week. Kim's regular earnings are $400 per week. His overtime earnings gave him an additional $60.

Total earnings = regular earnings + overtime

The formula can be written using letters to stand for the definitions.

$T = E + O$

T = total earnings
E = regular earnings
O = overtime

Solve the equation by plugging in numbers for the letters.

$T = \$400 + \$60$

$T = \$460$

You solved for the unknown value and found that the total earnings are $460.

Now Try These

a. Write a formula for the number of seconds in any number of hours.

Seconds (S) = # of hours (H) × 60 (min. per hour) × 60 (sec. per min.)
S = H × 3600

b. Write a formula for the number of days one employee works every year.

Workdays = (52 weeks × 5 days + overtime days) - vacation and sick leave

Some formulas require several math operations. You will need to know the order of operations to solve these more complex formulas.

Order of Operations

1. Do everything inside the parentheses first.
2. Do all multiplication and division in order going from left to right.
3. Do all addition and subtraction in order from left to right.

Example 3

Verna is joining a bowling league. Her average score is 120. Use the formula below to find Verna's bowling handicap.

Handicap = 0.8 (200 – average)

$H = 0.8(200 - a)$ Write the formula.

$H = 0.8(200 - 120)$ Substitute the values.

Solve for the unknown by using the order of operations

$H = 0.8(80)$ Parentheses first

$$\begin{array}{r} 80 \\ \underline{\times\, 0.8} \\ 64.0 \end{array}$$ Multiply

$H = 64$

Verna's handicap is 64 points.

Now Try These

a. Find Frank's bowling handicap if his average is 150.
Handicap = 0.8 (200 - average)

40 points

b. Find the Celsius temperature reading when the Fahrenheit temperature reading is 41°. Use the formula below where °C = Celsius temperature and °F = Fahrenheit temperature.

$$^{\circ}C = \frac{5}{9}\ (^{\circ}F - 32)$$

°C = 5 degrees

Skill Builders

1. Use the formula to find the distances traveled.

Distance = Rate × Time
$D = R \times T$

a. rate = 60 miles per hour
time = 3 hours ***D = 180 miles***

b. rate = 25 miles per hour
time = 0.5 hour ***D = 12.5 miles***

2. Find the etch rate for removing a metallic layer in each situation below.

$$\text{Etch Rate} = \frac{\text{thickness before - thickness after}}{\text{etch time}}$$

Etch time	*Thickness before etch*	*Thickness after etch*	*Etch rate*
a. 10 minutes	16.4 mm	10.2 mm	***0.62 mm*** per min.
b. 4 minutes	7.8 mm	6.8 mm	***0.25 mm*** per min.
c. 30 minutes	48.86 mm	33.26 mm	***0.52 mm*** per min.

3. Find the interest to be paid on the following loans.

Simple Interest = Principal × Rate × Time
I = PRT

Amount of Interest	*Principal*	*Rate*	*Time*
a. $ 3,200.00	$10,000.00	8%	4 years
b. $ 108.00	$600.00	18%	1 year

4. Find the percentage of computer parts that passed final inspection in each case below.

$$\text{Yield} = \frac{\text{quantity out}}{\text{quantity in}}$$

Change this to % form to get *percent yield.*
(round to the nearest tenth of a percent)

Quantity Out	*Quantity In*	*% Yield*
a. 420	448	93.8%
b. 1,086	1,200	90.5%

5. Compare the productivity of two employees working as stockkeepers. Use the productivity formula for nonselling employees to find which employee was more productive during a two-week period.

$$\text{Units processed per } \$1 = \frac{\text{units of work}}{\text{total pay to do work}}$$

Employee	*Processed Cases of Stock*	*Total Pay*
Keith	3,488	$640.00 = ***5.45 units***
Glenn	4,044	$800.00 = ***5.06 units***

Keith was more productive.

6. A car is purchased for $9,450. The credit union requires a 15% down payment. How much is the down payment, and how much will be financed?

Down Payment = Rate(%) × Whole

Down payment = $1,417.50
Amount financed = $8,032.50

7. Find the mortgage on a home when the purchase price is $65,000 and the down payment is 20% of the purchase price. Use both formulas given.

   a. Down payment = Rate(%) × Whole = ***$13,000.00***

   b. Purchase price – down payment = mortgage ***$52,000.00***

MEMO

TO: The Instructor
FROM: The Authors
SUBJECT: Graphing Ordered Pairs: Lesson 2.16

This lesson is intended to give learners a better understanding of graphing ordered pairs.

Vocabulary

Use these terms often during the group lesson to facilitate retention. The names of the axes and the terms *origin* and *coordinates* are especially important.

Develop Your Understanding

Many of the Now Try These and Skill Builders items require learners to plot points. Be sure to prepare transparencies or keys that show the solutions ahead of time.

Use transparencies or diagrams to clearly illustrate how to use both horizontal and vertical number lines to add and subtract (see figures 3 and 4). Remind learners that a thermometer is a good example of the applications of the vertical number line.

Skill Builders

Number 10 is a basic illustration of how mapping can be used to create diagrams.

Number 11 requires learners to apply number line addition and subtraction principles. It may help them draw in the new location of the figure. They should understand how to solve the problem both ways.

Learners work independently on this section. Have them check their work by using the Answer Key in the back of the book.

Expansion Activities

Bring in maps and/or blueprints to illustrate how coordinate grids are used.

Distribute blank grids, and have learners plot their own illustrations. They should then give a list of coordinates to another learner, who will plot and connect them.

Provide more practice examples for shifting the location of an object on a grid (similar to number 11 in Skill Builders).

Lesson 2.16
Graphing Ordered Pairs

Learning Objectives

To understand the concept of a coordinate system grid
To determine the coordinates of points on a grid
To graph points on a grid using coordinates

In the *Communications* book, Lesson 1.7, you will study three basic types of graphs that are frequently used in business. In this lesson, you will work with one additional type of graph, called *a coordinate system grid*. Coordinate system grids are used in drafting and mapping and also on some manufacturing machinery. Becoming familiar with coordinate system grids will assist you in reading blueprints, maps, and schematics more effectively.

Vocabulary

grid—a system of evenly spaced horizontal and vertical lines that represent units of measurement
coordinate system—a set of points located on a grid
x-axis—the horizontal base line of a grid
y-axis—the vertical base line of a grid
origin—the point where the x-axis intersects the y-axis
coordinates—the two numbers that show the location of a point on a grid
ordered pair—a set of coordinates in the form (x,y)
quadrant—any of the four parts of a coordinate system grid

Develop Your Understanding

Grids and Coordinate Systems

Technical diagrams, blueprints, and schematics must be very precise. Because of the need for precise measurement, these visual aids are usually made using a grid. A *grid* is a system of evenly spaced horizontal and vertical lines that represent units of measurement. Grids are used to locate or plot points. A set of points on a grid is called a *coordinate system*. An example of a grid is shown below.

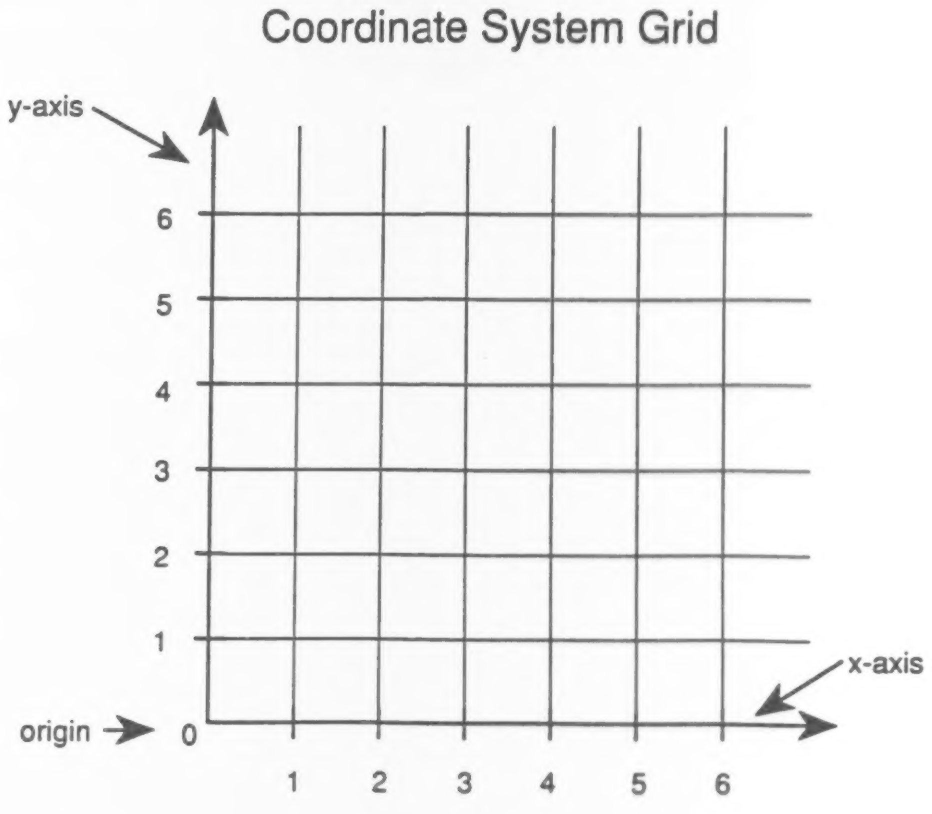

Coordinate System Grid
y-axis
x-axis
origin
0
1
2
3
4
5
6
1
2
3
4
5
6

Figure 1

The grid in figure 1 shows only positive values.

The horizontal line at the bottom is called the *x-axis*. The vertical line at the far left is called the *y-axis*. The arrows at the ends of the x-axis and the y-axis show that, in theory, these lines can extend infinitely. In practice, you can make a grid as large or as small as needed for your purpose.

The point where the x-axis meets the y-axis is called the *origin*.

To locate a point on a grid, you would need to know the point's *coordinates*. The coordinates of a point are the two numbers that show the point's horizontal and vertical locations. The horizontal coordinate, or *x*-value, is always given first. The vertical coordinate, or *y*-value, is always given second. The two coordinates together are called an *ordered pair*. Ordered pairs are written in the form (x,y), with numbers plugged in to replace the letters. For example, the coordinates of the origin are (0,0).

Example 1

Find the point indicated by the ordered pair (3,2).

In the ordered pair (3,2), 3 is the x-coordinate, and 2 is the y-coordinate.

1. Start at the origin, and count 3 places to the right along the x-axis. This is the location of the *x*-value 3.
2. Count 2 units up from the *x*-axis point 3.
3. You should now be at the intersection of the lines at coordinate 3 on the *x*-axis and coordinate 2 on the *y*-axis, which is the point (3,2).

The point (3,2) is marked on the grid in figure 2 with the letter "A."

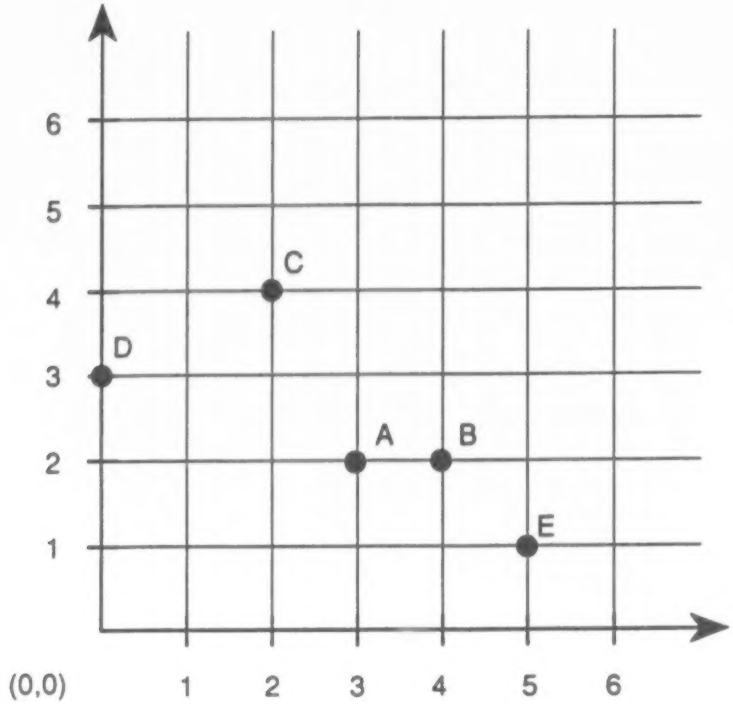

6
5
4
3
2
1
(0,0)
1
2
3
4
5
6
C
D
A
B
E

Figure 2

Now Try These

Look at figure 2, and fill in the blanks with the letters that mark these points:

a. (2,4) *C*

b. (4,2) *B*

c. (5,1) *E*

d. (0,3) *D*

Four-Quadrant Coordinate Systems

Many coordinate system grids use both positive and negative numbers. This type of grid, like the ones you have already used, includes an *x*-axis and a *y*-axis that cross at the origin. This time, however, there are arrows at both ends of each axis, to show that they can be extended infinitely in both directions.

The *x*-axis in this type of system looks like a horizontal number line:

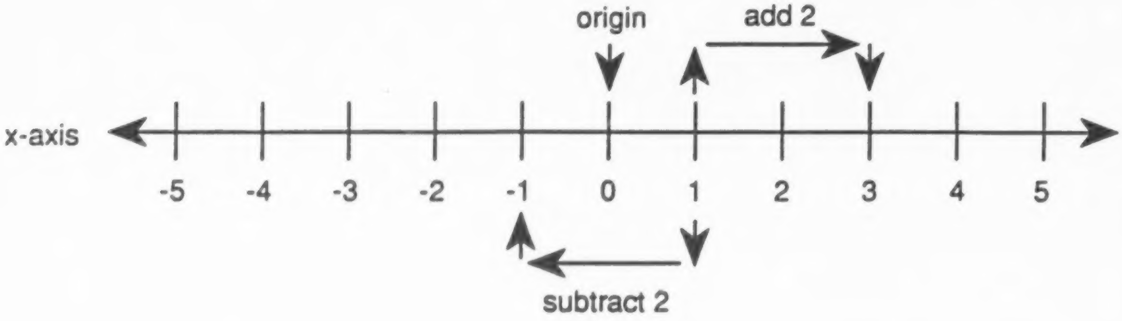


Figure 3

Note that integers are evenly spaced along the *x*-axis. Positive numbers are always on the right of the origin (the point 0), while negative numbers are always on the left. Moving along the axis to the right is like adding. For example, if you start at the *x*-value 1 and move 2 units to the right, you will arrive at the *x*-value 3.

This is the same as adding:

$1 + 2 = 3$

On the other hand, moving along the axis to the left is like subtracting. If you start at the *x*-value 1 and move 2 units to the left, you will arrive at the *x*-value -1.

This is the same as subtracting:

$1 - 2 = -1$

The *y*-axis works in the same way, except that this number line is vertical.

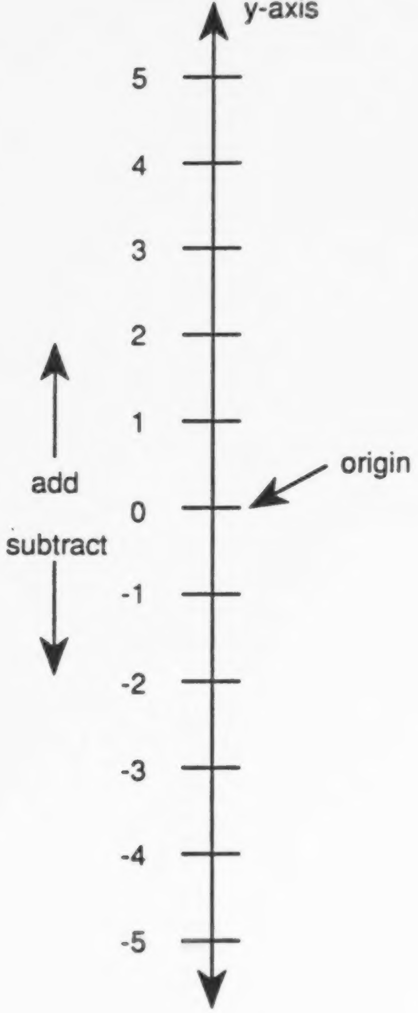

y-axis
5
4
3
2
1
origin
add
0
subtract
-1
-2
-3
-4
-5

Figure 4

On the *y*-axis, values above the origin are positive, and values below the origin are negative. Therefore, moving up is the same as adding, and moving down is the same as subtracting.

An example of a complete coordinate system grid is shown in figure 5.

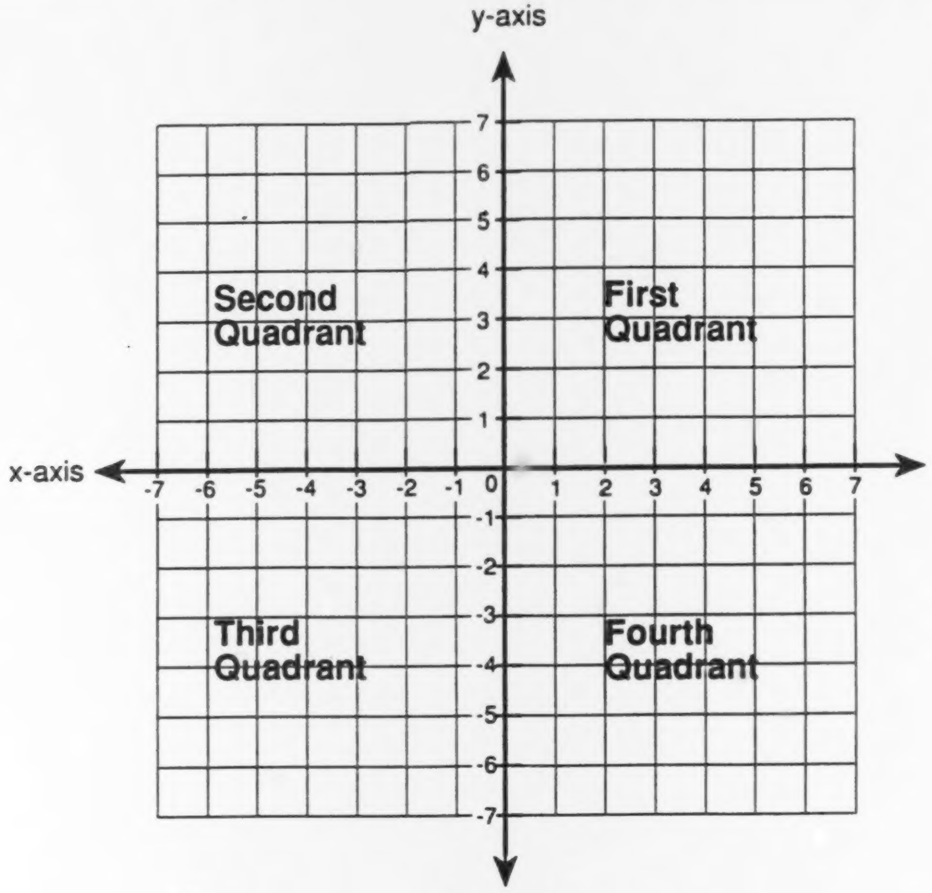

y-axis
x-axis
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7
Second Quadrant
First Quadrant
Third Quadrant
Fourth Quadrant

Figure 5

Note that this grid is divided into four quadrants:

First Quadrant:	Positive *x*-values and positive *y*-values
Second Quadrant:	Negative *x*-values and positive *y*-values
Third Quadrant:	Negative *x*-values and negative *y*-values
Fourth Quadrant:	Positive *x*-values and negative *y*-values

The procedure for locating points on this type of grid is the same as the procedure you practiced in the previous section.

Example 2

Find the point indicated by the ordered pair (-1,4).

In the ordered pair (-1,4), -1 is the *x*-coordinate, and 4 is the *y*-coordinate.

1. Starting at the origin, find the point -1 on the *x*-axis by counting one unit to the left.
2. From that point, count 4 units straight up.
3. You should now be at the point where the lines extending from the *x*-value -1 and the *y*-value 4 cross. This is the point (-1, 4)

The point (-1, 4) is marked on the grid in figure 6 with the letter "F."

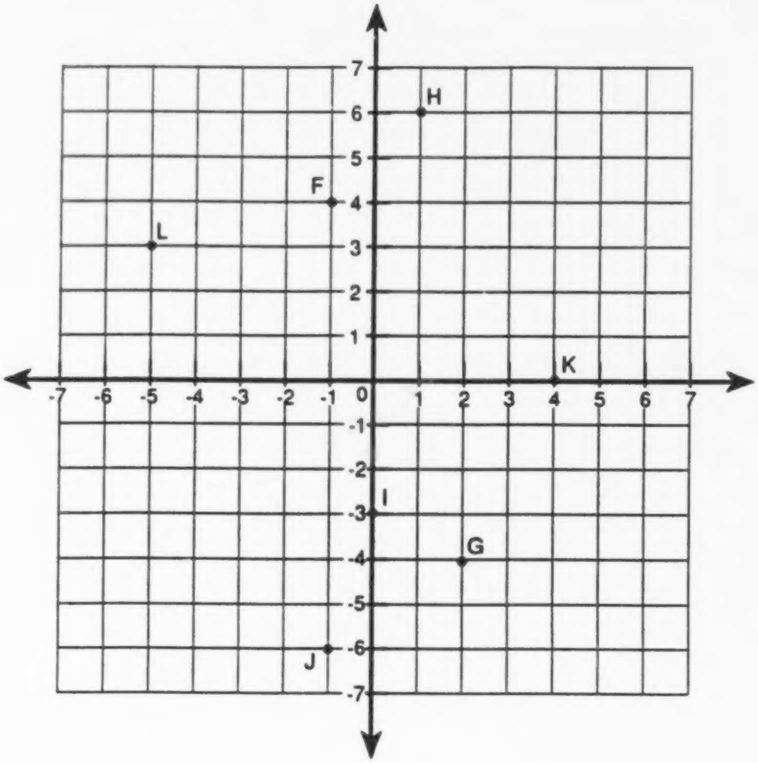


Figure 6

Now Try These

Look at figure 6, and fill in the blanks with the letters that mark these points:

a. (1,6) ___*H*___ b. (-1,-6) ___*J*___ c. (2,-4) ___*G*___

d. (-5,3) ___*L*___ e. (0,-3) ___*I*___ f. (4,0) ___*K*___

Plotting Points on a Grid

You can use coordinates to plot points on a grid or coordinate system.

Example 3

Plot the point (-2,-6) on the grid in figure 7.

In the ordered pair (-2,-6), -2 is the x-coordinate, and -6 is the y-coordinate.

1. Start at the origin. Count 2 units to the left on the x-axis. This point has the x-value of -2.
2. Count down six units from this point. You should be at the place where the lines coming from the x-value -2 and the y-value -6 intersect.
3. Mark this point with a dot, and label it (-2,-6).

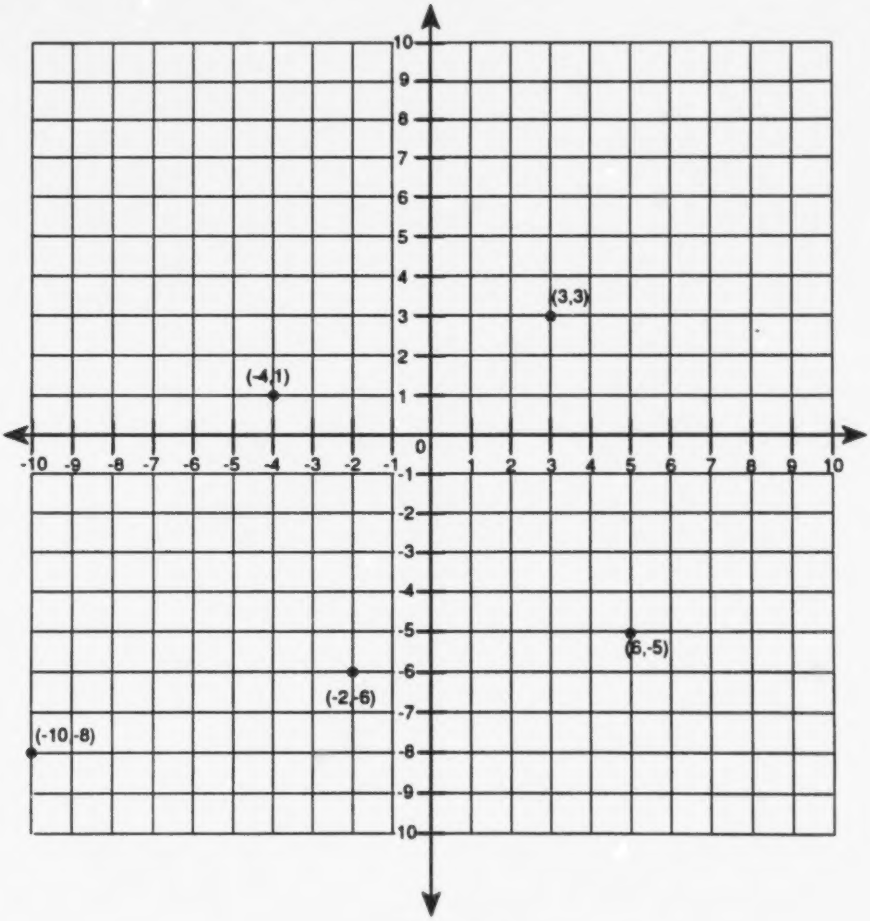


Figure 7

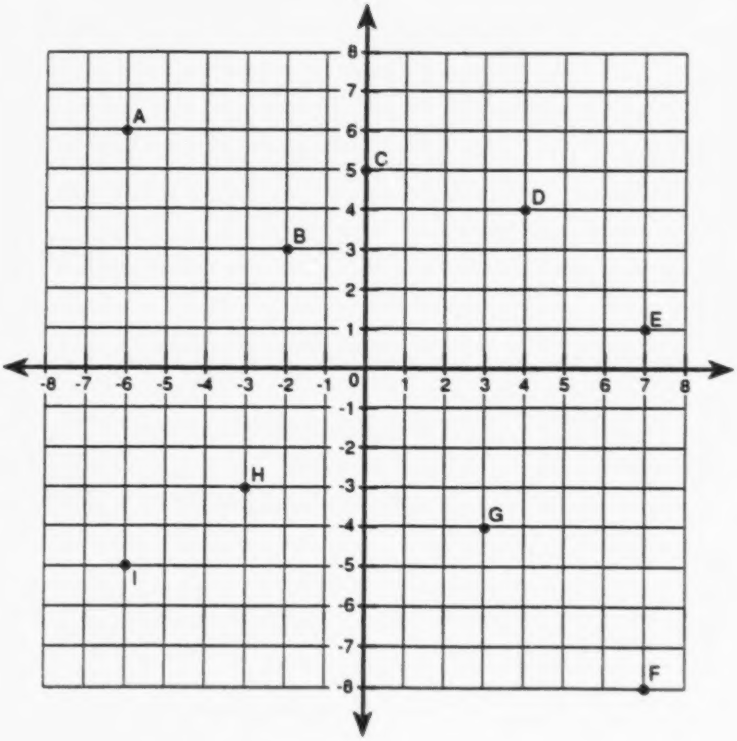
Now Try These

Find and plot each of these points on the grid in figure 7.

a. (-4,1) b. (3,3) c. (-10,-8) d. (6,-5)

Skill Builders

Using the grid below, find the coordinates of each point, and write them next to the letter corresponding to the point.


A
B
C
D
E
F
G
H
I
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-8 -7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7 8

1. A *(-6,6)*

2. B *(-2,3)*

3. C *(0,5)*

4. D *(4,4)* 5. E *(7,1)* 6. F *(7,-8)*

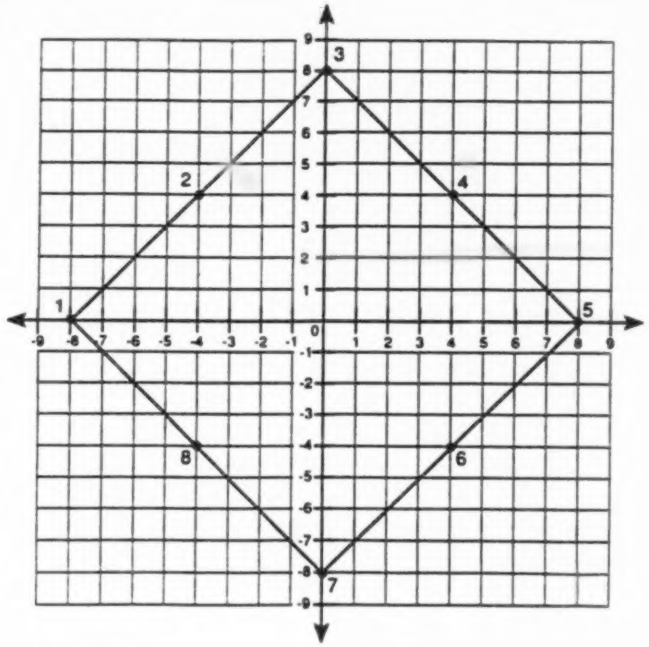
7. G *(3,-4)* 8. H *(-3,-3)* 9. I *(-6,-5)*

10. Plot each of the following points on the grid below. Then, connect the points in order to make a drawing.

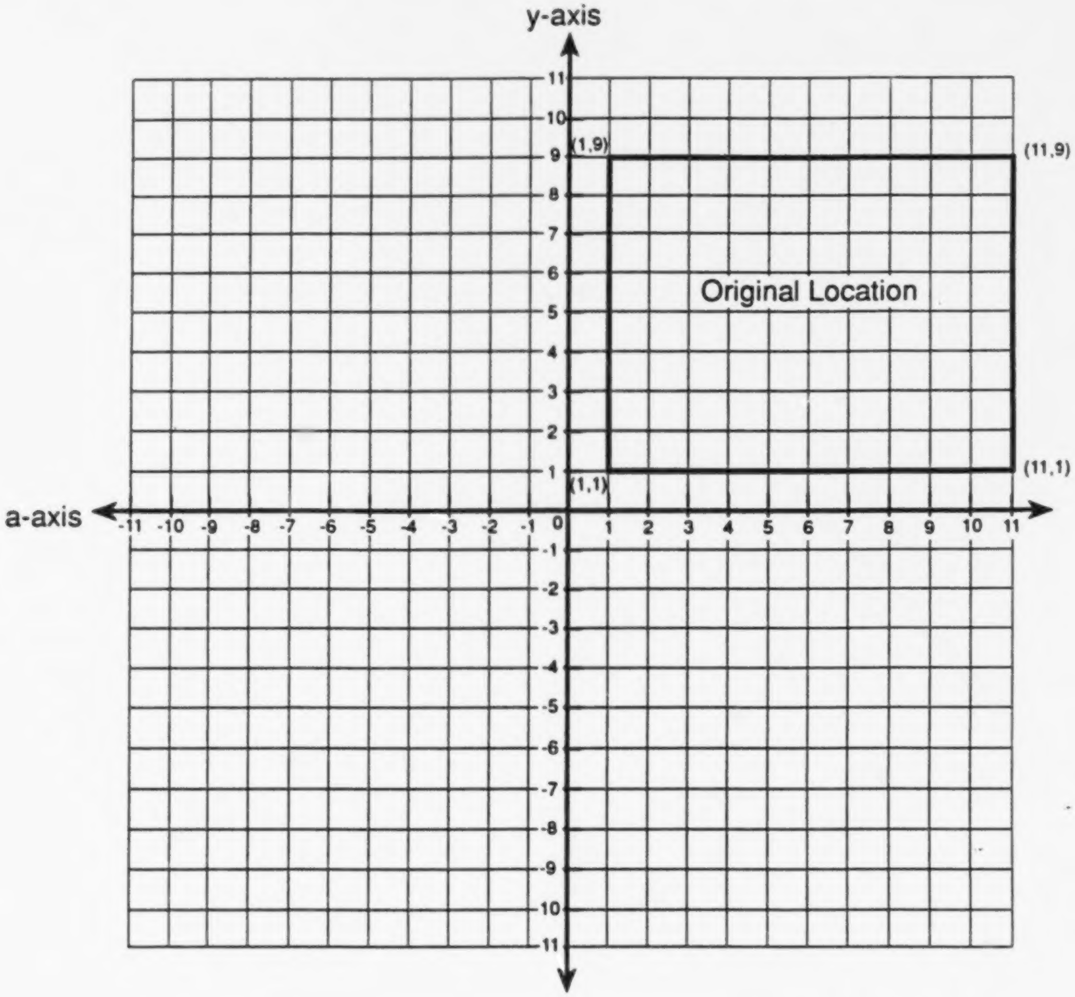
Point 1: (-8,0)
Point 2: (-4,4)
Point 3: (0,8)
Point 4: (4,4)

Point 5: (8,0)
Point 6: (4,-4)
Point 7: (0,-8)
Point 8: (-4,-4)

(After connecting all the points in order, connect point 8 back to point 1.)



11. The diagram below shows the layout of a rectangular figure on a grid. Each unit on the grid represents one inch.


y-axis
a-axis
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11
(1,9)
(11,9)
Original Location
(1,1)
(11,1)

The original coordinates of the 4 corners of this figure are:

Lower left corner:	(1,1)
Upper left corner:	(1,9)
Lower right corner:	(11,1)
Upper right corner:	(11,9)

After the original drawing was made, the rectangle was moved 3 inches to the left and 2 inches down. What are the new coordinates of:

a. the lower left corner? ***(-2,-1)***

b. the upper left corner? ***(-2,7)***

c. the lower right corner? ***(8,-1)***

d. the upper right corner? ***(8,7)***

(Hint: To solve this problem, you can either subtract 3 from each x-value and 2 from each y-value or use the grid to count out the adjustments for each ordered pair.)

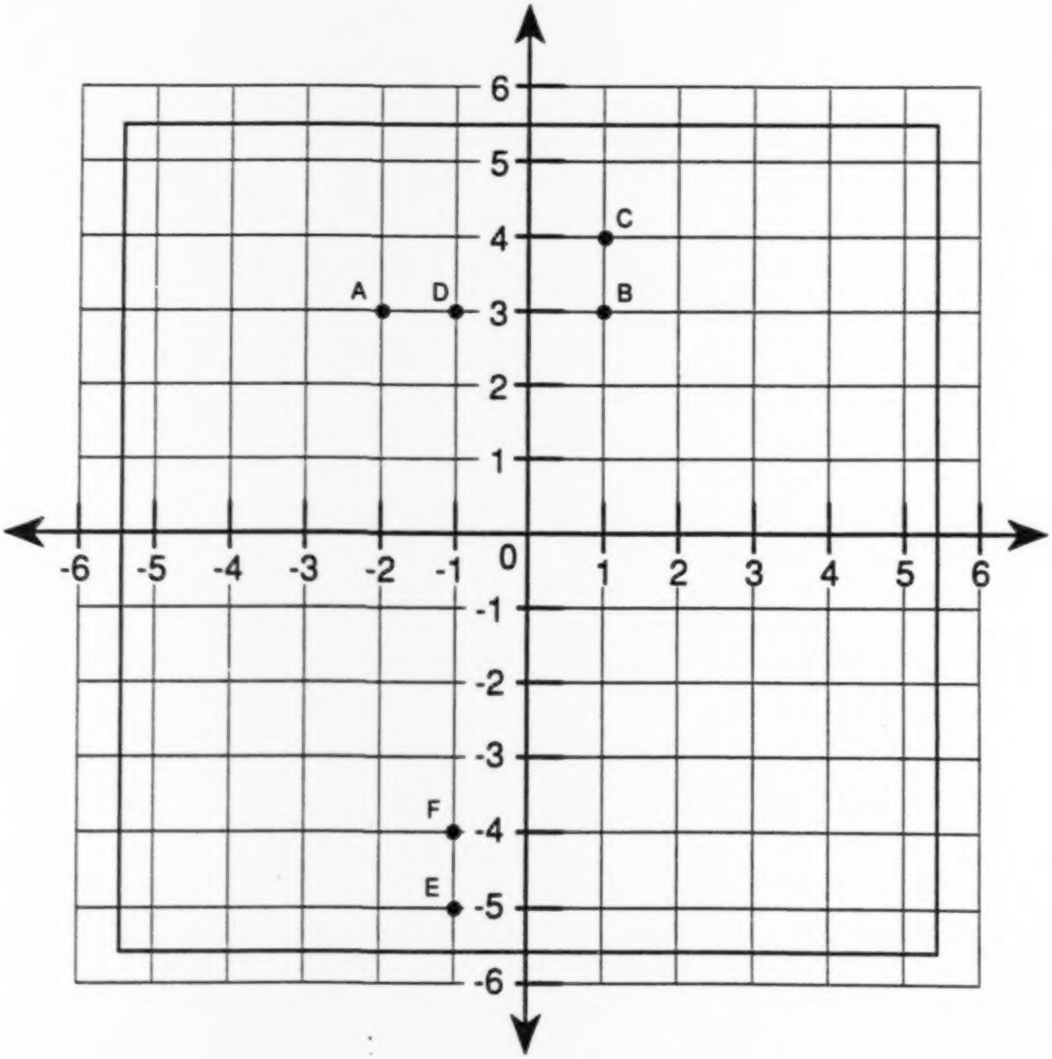
12. The diagram below shows a computer wiring board on a grid. The lines in the grid are 1 millimeter apart. You must drill three sets of holes in the board according to the following instructions:

   a. Drill Hole A at (-2,3) and Hole B exactly 3 mm to the right of Hole A.

   b. Drill Hole C at (1,4) and Hole D exactly 2 mm to the left and 1 mm below Hole C.

   c. Drill Hole E at (-1,-5) and Hole F exactly 1 mm above Hole E.

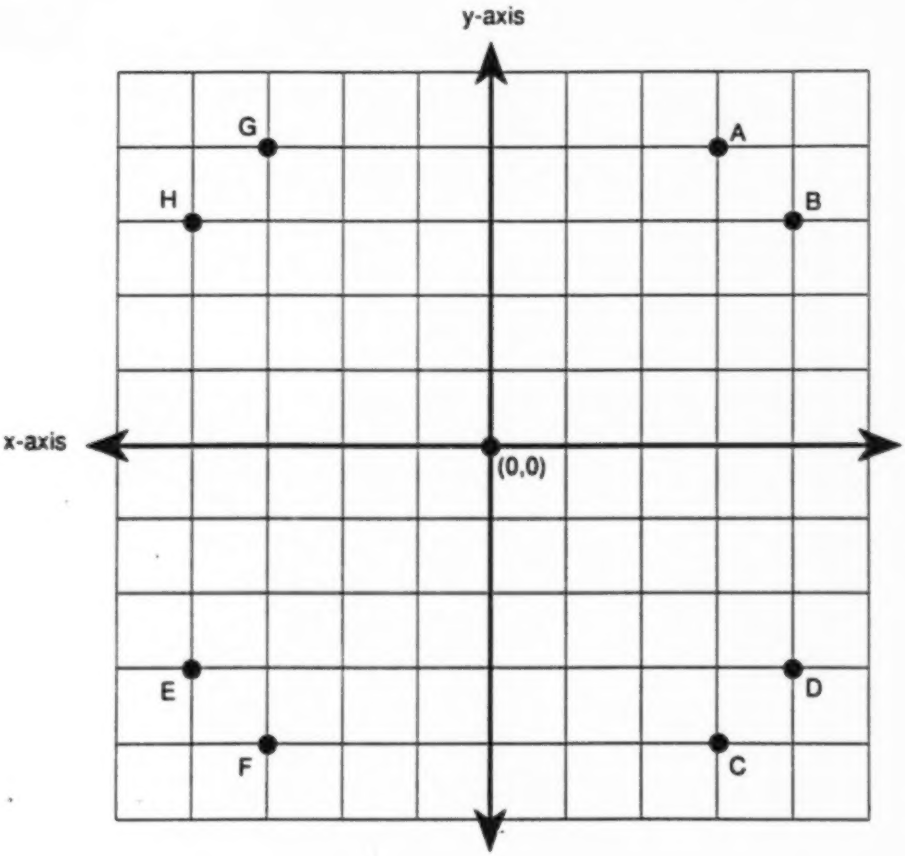
Make a dot at the location of each hole in the board, and label the dots with the correct letters according to the instructions above.



Check Yourself

Which of the points shown below have positive *x*-values?

A, B, C, and D


y-axis
x-axis
G
A
H
B
(0,0)
E
D
F
C

Module 2 Review #3
Lessons 2.12-2.16

You may wish to review the key concepts in Lessons 2.12 through 2.16 before completing these review exercises. However, if you feel comfortable with what you have learned, go ahead and attempt them. You may keep your lessons at hand to use as a reference source, but try to solve each problem without looking back at the lessons.

Solve each of the following:

1. $3 + 2 \times 4 - 5 =$ ***6***

2. $4 + (3 \times 5) - 8 =$ ***11***

3. Last month Company A produced 10 times as many widgets as it did gizmos. If the total produced was 110, how many gizmos and how many widgets were produced?

   10 gizmos and 100 widgets

Show all work for the following equations:

4. $4x + 5 = ?$ where $x = 3$ ***17***

5. $6 + N = 28$

   N = 22

6. $B - 7 = 3$
   B = 10

7. $8X = 16$
   X = 2

8. $9C = 45$
   C = 5

9. $\frac{9}{X} = 3$
   X = 3

10. $\frac{8}{X} = 2$

$X = 4$

11. $\frac{1}{5} X = 6$

$X = 30$

12. $\frac{4}{5} X = 8$

$X = 10$

13. $\frac{X}{3} = 9$

$X = 27$

14. $\frac{X}{5} = 20$

$X = 100$

15. $X + 12 = 20$

$X = 8$

16. $14 + X = 3$

$X = -11$

17. $5 = R - 10$

$R = 15$

18. $12 = N - 12$

$N = 24$

19. John produced four times as many panels last week as Bob. If the total they produced was 70, how many did John produce and how many did Bob produce?

Bob produced 14.
John produced 56.

20. If a certain number is divided by 5 and gives an answer of 5, what is the number?

25

21. Use this formula to figure the following:

I = PRT where I = interest

P = principal

R = rate of interest

T = time of loan

John takes out a car loan at 12% for 5 years for a total of $9,000. How much total interest will he pay?

$5,400

22. Find the distances traveled if:

Distance = Rate × Time

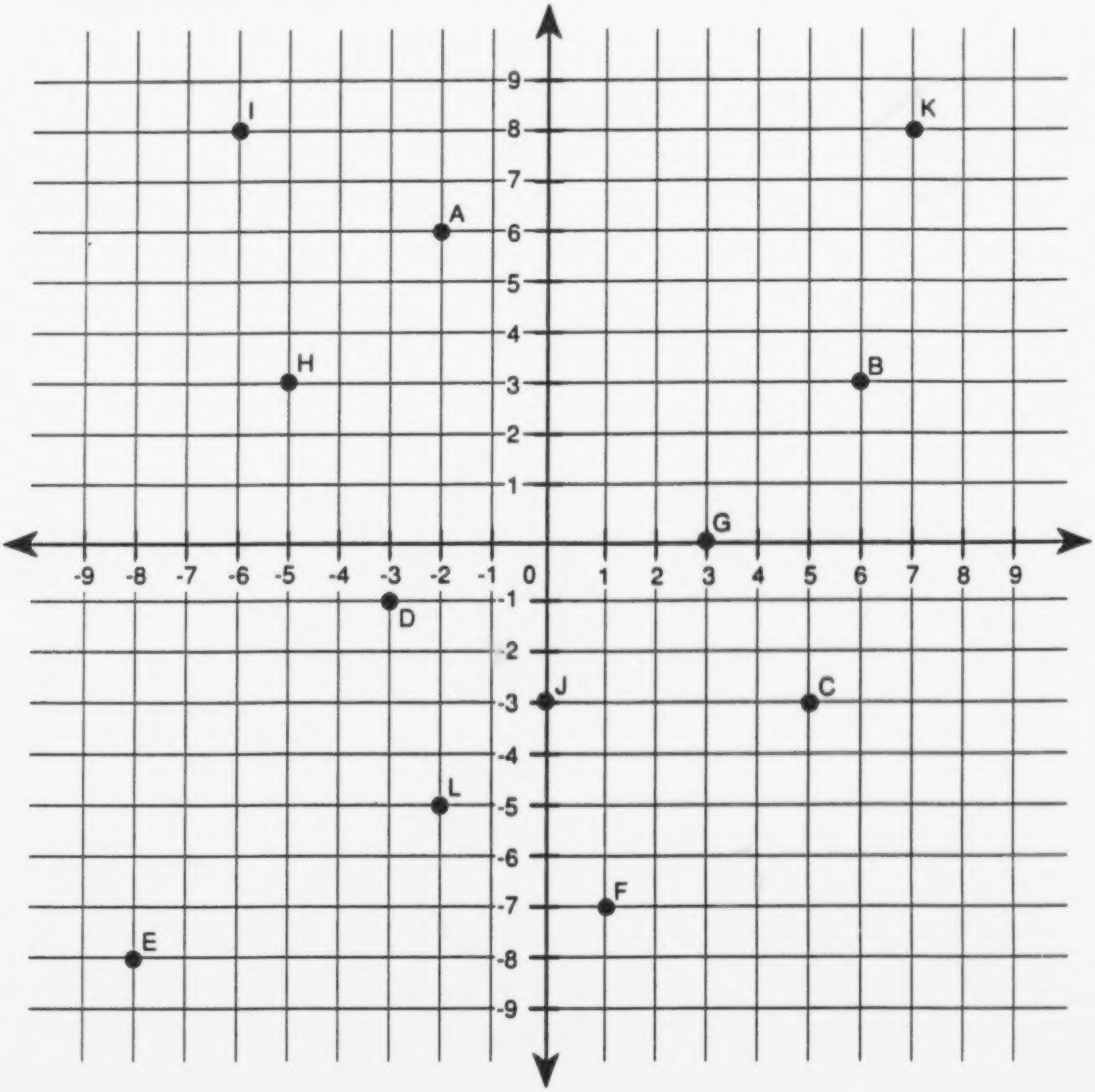
A. Rate = 65 miles per hour Time = 3.5 hours

227.5 miles

B. Rate = 30 miles per hour Time = 7.4 hours

222 miles

23. Refer to the following grid, and identify the coordinates of the points marked on it.


I
K
A
H
B
G
D
J
C
L
F
E
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9

Point A: ***(-2,6)***

B: ***(6,3)***

C: ***(5,-3)***

D: ***(-3,-1)***

E: ***(-8,-8)***

F: ***(1,-7)***

G: ***(3,0)***

H: ***(-5,3)***

I: ***(-6,8)***

J: ***(0,-3)***

K: ***(7,8)***

L: ***(-2,-5)***

NAME ______________________________

Module 2 Posttest

Directions: *Do your calculations in the space provided.*

1. You have been asked to buy some machine parts. The cost of each part is listed below. Estimate the total cost of the parts, rounding to the nearest whole dollar. (Your answer should be an *estimate*, not the actual total!)

   $25.75
   $10.33
   $12.50
   $11.29
   $17.68

2. Pat was told to cut a 42" cord into eight 5" pieces. How many inches of the cord will be left after the eight pieces are cut?

3. Two thirds ($\frac{2}{3}$) of the employees at ABC Computer Corporation gave blood this year. 2,430 people work for ABC. How many of these employees gave blood?

4. A recipe for mixing paste calls for $\frac{1}{2}$ cup of water to make $\frac{1}{3}$ jar of paste. How much water would you use to make 2 full jars of paste?

5. Production Area A completed the following numbers of jobs on each work day last week. What was the area's average daily number of jobs completed for the week? DO NOT round your answer.

Monday:	150 jobs
Tuesday:	200 jobs
Wednesday:	175 jobs
Thursday:	125 jobs
Friday:	178 jobs

6. For the first six months of 1991, Robert's number of days absent per month is shown below. What was his average number of days absent per month? (Round your answer to the nearest hundredth.)

Month	Days Absent
January	10
February	0
March	2
April	0
May	3
June	1

7. What is the value of N?

$$\frac{N}{24} = \frac{3}{8}$$

8. In order to make Lovely Orange paint, you must mix the following colors:

4 parts red
3 parts yellow
2 parts white

How many jars of red paint would you need if you used 6 jars of white?

9. If you can get exactly 14 gallons of gas for $21.00, how much gas could you buy with $12.00?

10. What is 6% of 102?

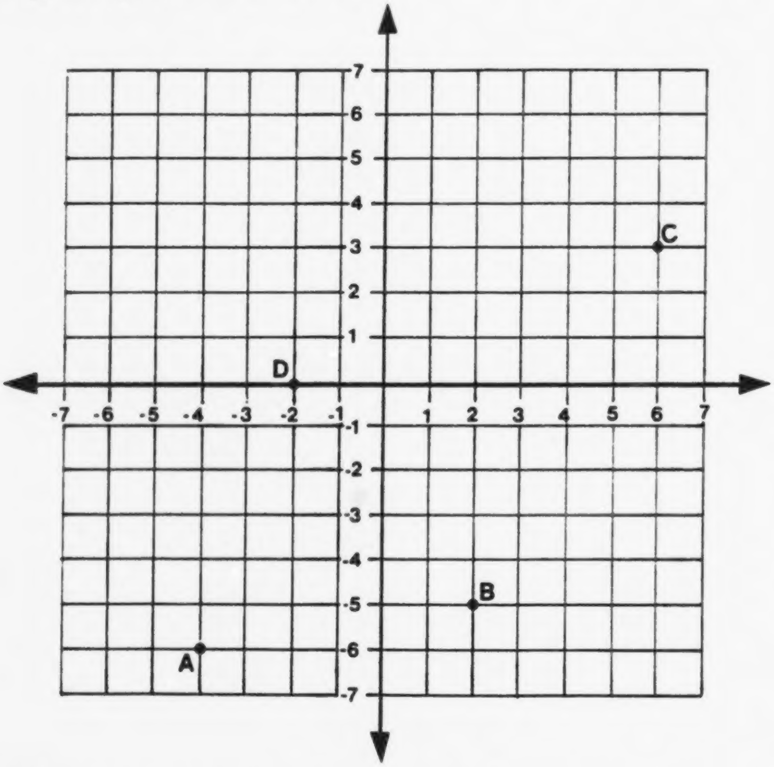
11. Express this fraction as a percentage:

$$\frac{4}{25} = ?\%$$

12. 96% of a total of 1,025 computer parts passed inspection. How many parts passed inspection?

13. A publishing company spends $200 out of every $500 of its expenses on employee payroll and benefits. What percentage of its expenses goes toward payroll and benefits?

Use the coordinate system grid shown below to answer questions 14 and 15. Each unit on the grid represents one inch.


7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6
7
A
B
C
D

14. What are the coordinates of each of the following points?

A _______

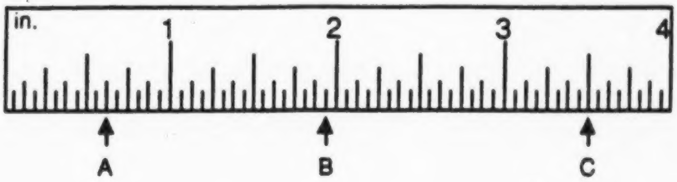
B _______

C _______

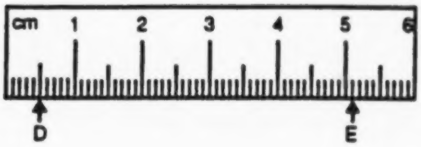
D _______

15. If point A were moved 2 inches to the left, what would be its new coordinates? (Remember, each unit on the grid is one inch.)

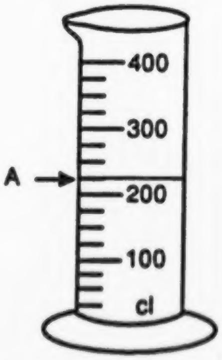
16. Using the *standard* ruler shown below, what measurements are marked by the letters A, B, and C?



17. Using the *metric* ruler below, what is the distance between point D and point E?


cm
1
2
3
4
5
6
D
E

18. How much liquid is in the cylinder shown below (filled to the line marked "A")?


400
300
A
200
100
cl

19. What is the perimeter of a square panel if the length of one side is 4 inches?

20. If 1 centimeter = 0.39 inch, how many inches are in 12 centimeters?

21. $(-2) + (-4) =$

22. $30 \times (-4) =$

23. $7 - (-3) =$

24. What is the value of N?

$N - 12 = 18$

25. Last month, ABC Computers produced three times as many laptop computers as it did standard personal computers. The combined number of laptop and standard personal computers produced was 20,400. How many *standard personal computers* were produced?

26. What is the value of x?

$$9x = (-27)$$

27. What is the value of b?

$$\frac{b}{9} = 10$$

28. When a certain number is divided by 9, the quotient is also 9. What is the number?

29. Use this formula for finding distance, to calculate the miles traveled on a business trip.

Distance = Rate × Time

where Rate = 55 miles per hour
Time = 6 hours

30. Use this formula to calculate percentage yield for computer parts. Your answer must be a percent.

$$\text{Percentage Yield} = \frac{\text{quantity out}}{\text{quantity in}}$$

where quantity out = 76
quantity in = 80

Module 1 Pretest
Answer Key

1. d. 15.563
2. c. 6.25
3. 6
4. 6.04
5. 65.451
6. 12.6
7. 2.017
8. 9860.4
9. 0.1277
10. 1.25
11. 12,580
12. $4\frac{3}{8}$
13. $\frac{10}{7}$
14. $\frac{4}{5}$
15. $1\frac{1}{7}$
16. $\frac{1}{3}$
17. $\frac{1}{3}$
18. 12
19. 8
20. $\frac{19}{20}$
21. 16
22. $\frac{5}{9}$
23. $1\frac{4}{15}$
24. 2
25. $\frac{3}{8}$
26. 2
27. $3\frac{3}{4}$

Module 1 Posttest
Answer Key

1. c. 9.794
2. a 11.99
3. 5
4. 25.846
5. 0.02706
6. 0.136
7. 2.568
8. 126
9. 0.9381
10. 4.8
11. 6420
12. $6\frac{2}{9}$
13. $\frac{26}{3}$
14. $\frac{3}{5}$
15. $1\frac{1}{4}$
16. $\frac{1}{2}$
17. $\frac{1}{6}$
18. 12
19. 15
20. $\frac{13}{18}$
21. $18\frac{7}{8}$
22. $\frac{3}{5}$
23. $1\frac{1}{12}$
24. 3
25. $\frac{1}{8}$
26. $7\frac{1}{2}$
27. $5\frac{3}{5}$

Module 2 Pretest
Answer Key

1. $108.00
2. $166
3. 900 employees
4. 5
5. 3 cups
6. 16
7. 84
8. 20 inches
9. 8 jars
10. 10 gallons
11. 22.4
12. 15%
13. 65 boards
14. 15%
15. A: (−3, −5) B: (3, −4) C: (5, 5) D: (−2, 1)
16. (3,5)
17. A: $\frac{1}{2}$ inch
    B: $2\frac{1}{16}$ inches
    C: $3\frac{1}{4}$ inches
18. 2.5 cm or 2 cm 5 mm
19. 150 cl
20. 28 mm
21. -2
22. -42
23. 9
24. 85
25. -2
26. 7
27. 64
28. 12
29. 360 miles
30. 95%

Module 2 Posttest
Answer Key

1. $78.00
2. 2 inches
3. 1,620
4. 3 cups
5. 165.6 jobs
6. 2.67 days
7. N = 9
8. 12 jars
9. 8 gallons
10. 6.12
11. 16%
12. 984
13. 40%
14. A: (-4,-6) B: (2,-5) C: (6,3) D: (-2, 0)
15. (-6,-6)
16. A: $\frac{5}{8}$ inch B: $1\frac{15}{16}$ inches C: $3\frac{1}{2}$ inches
17. 4.6 cm or 4 cm 6 mm
18. 225 cl
19. 16 inches
20. 4.68 inches
21. -6
22. -120
23. 10
24. N = 30
25. 5,100 standard personal computers
26. x = -3
27. b = 90
28. 81
29. 330 miles
30. 95%

SN 150 I